Testing Distributed Systems through Symbolic Model Checking

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Need for Validation

Distributed control systems

- concurrent processes
- running on physically distributed hardware
- hard to design in nature
- critical systems (e.g. plant control system, ...)

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Validation Techniques

Model-Checking:

- exhaustive verification
- but: not (yet) scalable to real-sized systems
- works on a model of the system

Testing and Monitoring:

- non-exhaustive verification
- scalable for real-sized systems
- widely used in industry
- works on the real system

Testing

The System

- distributed and asynchronous
- instrumented to emit relevant events
  (e.g. variable assignments, message transfer)
- execution traces are collected

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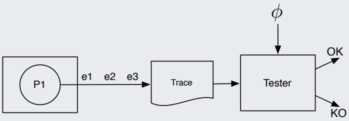
The Tester

- analyses the collected traces
- checks whether a certain property holds

Centralized v.s. Distributed Systems

Centralized System

- only one process
- events are totally ordered



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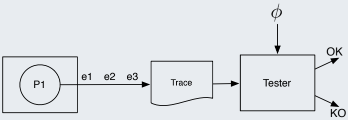
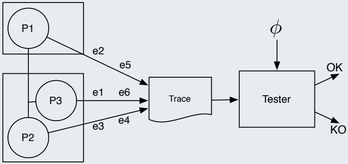


Distributed System

- multiple processes
- events are not totally ordered
- but a partial order can be obtained using vector clocks [Lam78, Mat89]



Partial Order Traces

Model

- partially ordered set of events
- events are labelled with assignments
- two restrictions:
  1. two events of the same process must be ordered
  2. two events assigning the same variable must be ordered

$$\begin{tikzcd} P_1 & x{:=}1 \arrow[r] & y{:=}3 \arrow[r] \arrow[d] & z{:=}4 \\ P_2 & w{:=}4 \arrow[r] & x{:=}0 & \end{tikzcd}$$

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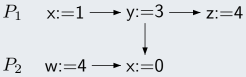
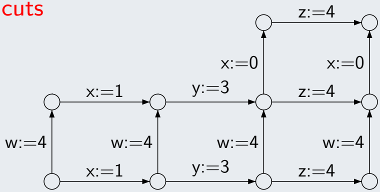


Semantics

- a lattice of configurations called cuts



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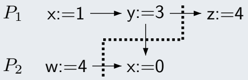
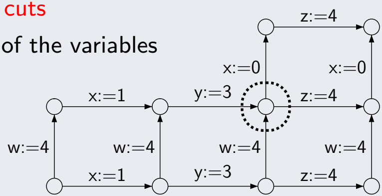


Semantics

- a lattice of configurations called cuts
- each cut has a unique valuation of the variables

$$v : \begin{cases} w & \mapsto & 4 \\ x & \mapsto & 1 \\ y & \mapsto & 3 \\ z & \mapsto & 0 \end{cases}$$



Global Predicate Detection

So far...

Efficient techniques have been developed for several classes of predicate:

- Stable predicates [CL85] (such that $\phi \implies \mathrm{AG}\phi$)
- Disjunctive predicates (of the form $\mathrm{EF}(p_1 \vee p_2 \vee \cdots \vee p_n)$)
- Conjunctive predicates [GW94, GW96] (of the form $\mathrm{EF}(p_1 \wedge p_2 \wedge \cdots \wedge p_n)$)
- Observer independent predicates [CDF95] (such that $\mathrm{AF}\phi \iff \mathrm{EF}\phi$)
- Linear, semi-linear predicates [CG98]
- Regular predicates (RCTL logic) [GM01, SG03]

$$\phi ::= \top \mid p \mid \neg p \mid \phi \wedge \phi \mid \mathrm{EF}\phi \mid \mathrm{EG}\phi \mid AG\phi \mid \mathrm{EX}[i]\phi$$

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Our goal

Provide an efficient technique for full CTL (Computational Tree Logic):

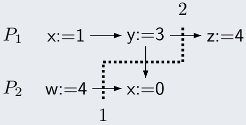
$$\phi ::= \top \mid p \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \mathrm{EX}\phi \mid \mathrm{AX}\phi \mid \mathrm{EG}\phi \mid \mathrm{AG}\phi \mid \mathrm{E}[\phi \mathrm{U} \phi] \mid \mathrm{A}[\phi \mathrm{U} \phi]$$

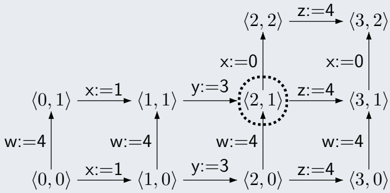
Symbolic Representation

Multirectangles

Cuts can be viewed as tuples of naturals:

- in a tuple $\langle x_1, \ldots, x_n \rangle$, $x_i =$ the number of events process i has executed





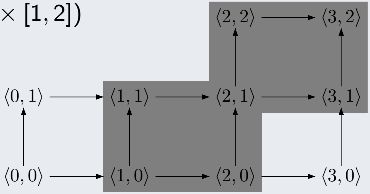
Symbolic Representation

Multirectangles

Cuts can be viewed as tuples of naturals:

- in a tuple $\langle x_1, \ldots, x_n \rangle$, $x_i =$ the number of events process i has executed
- sets of cut can be represented as a union of multirectangles

$$= ([1,2] \times [0,1]) \cup ([2,3] \times [1,2])$$



Symbolic Representation (cont'd)

Interval Sharing Tree (IST)

Symbolic data structure for representing sets of tuples:

- directed acyclic graph
- nodes are labelled with intervals
- each path from $\top$ to $\perp$ encodes a multirectangle

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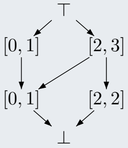

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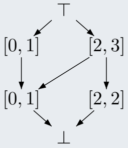

Using IST for CTL

For each CTL formula ϕ, we build symbolically an IST $\mathcal{I}_\phi$ representing $[\![\phi]\!]$

Symbolic Computation

Tautology

If $\phi = \top$, we build $\mathcal{I}_\top$ iteratively:

$$
\begin{tikzcd}
\text{x:=1} \arrow[r] & \text{y:=3} \arrow[r] \arrow[d] & \text{z:=4} \\
\text{w:=4} \arrow[r] & \text{x:=0} &
\end{tikzcd}
$$

Symbolic Computation

Tautology

If $\phi = \top$, we build $\mathcal{I}_\top$ iteratively:

- start as if the trace had no communications

x:=1 $\longrightarrow$ y:=3 $\longrightarrow$ z:=4

w:=4 $\longrightarrow$ x:=0

$$\top \to [0,3] \to [0,2] \to \bot$$

$\mathcal{I}_0$

Symbolic Computation

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If $\phi = \top$, we build $\mathcal{I}_\top$ iteratively:

- start as if the trace had no communications
- treat communications one at a time

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\end{tikzcd}$$

$$\underbrace{\begin{array}{c}\top\\ \downarrow\\ [0,3]\\ \downarrow\\ [0,2]\\ \downarrow\\ \bot\end{array}}_{\mathcal{I}_0} \setminus \left(\underbrace{\begin{array}{c}\top\\ \downarrow\\ [0,1]\\ \downarrow\\ [0,2]\\ \downarrow\\ \bot\end{array}}_{\mathcal{B}(\mathsf{y} := 3)} \bigcap \underbrace{\begin{array}{c}\top\\ \downarrow\\ [0,3]\\ \downarrow\\ [2,2]\\ \downarrow\\ \bot\end{array}}_{\mathcal{A}(\mathsf{x} := 0)} \right)$$

Symbolic Computation

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If $\phi = \top$, we build $\mathcal{I}_\top$ iteratively:

- start as if the trace had no communications
- treat communications one at a time
- stop when all communications have been treated

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$$\begin{tikzcd}[row sep=small] \top \arrow[d] \\ {[0,3]} \arrow[d] \\ {[0,2]} \arrow[d] \\ \bot \end{tikzcd} \setminus \left(\begin{tikzcd}[row sep=small] \top \arrow[d] \\ {[0,1]} \arrow[d] \\ {[0,2]} \arrow[d] \\ \bot \end{tikzcd} \cap \begin{tikzcd}[row sep=small] \top \arrow[d] \\ {[0,3]} \arrow[d] \\ {[2,2]} \arrow[d] \\ \bot \end{tikzcd} \right) = \begin{tikzcd}[row sep=small] & \top \arrow[dl] \arrow[dr] & \\ {[0,1]} \arrow[d] & & {[2,3]} \arrow[d] \arrow[dll] \\ {[0,1]} \arrow[dr] & & {[2,2]} \arrow[dl] \\ & \bot & \end{tikzcd}$$

$\mathcal{I}_0$ $\quad$ $\mathcal{B}(\text{y} := 3)$ $\quad$ $\mathcal{A}(\text{x} := 0)$ $\quad$ $\mathcal{I}_\top$

Symbolic Computation (cont'd)

Predicate

If $\phi = p \equiv (x \bullet c)$ with $\bullet \in \{=, \neq, <, \leq, >, \geq\}$, we build $\mathcal{I}_p$ as follows:

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$$p(x \neq 0)$$

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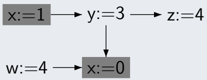

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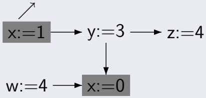
- we know that all events assigning x are totaly ordered
- some events sets p to true



$p(x \neq 0)$

$$\top \to [1,3] \to [0,2] \to \bot$$

$\mathcal{A}(\mathrm{x} := 1)$

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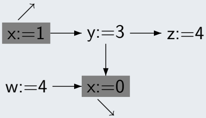
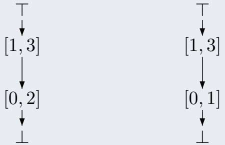


$p \equiv (x \neq 0)$



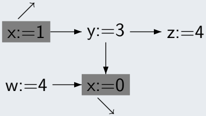
$\mathcal{A}(\text{x} := 1)$ $\mathcal{B}(\text{x} := 0)$

Symbolic Computation (cont'd)

Predicate

If $\phi = p \equiv (x \bullet c)$ with $\bullet \in \{=, \neq, <, \leq, >, \geq\}$, we build $\mathcal{I}_p$ as follows:

- we know that all events assigning x are totaly ordered
- some events sets p to true
- some events sets p to false
- compute the union of all slices where:
  - p was set to true,
  - but not yet to false



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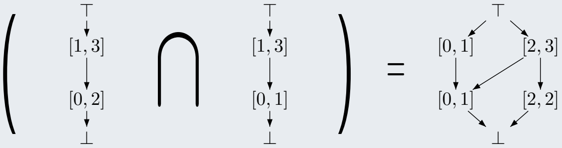

$\mathcal{A}(x := 1)$ $\mathcal{B}(x := 0)$ $\mathcal{I}_p$

Symbolic Computation (cont'd)

Boolean Operators

We can use standard set operations on IST:

$$
\begin{aligned}
[\![\phi_1 \wedge \phi_2]\!] &= [\![\phi_1]\!] \cap [\![\phi_2]\!] \\
[\![\phi_1 \vee \phi_2]\!] &= [\![\phi_1]\!] \cup [\![\phi_2]\!] \\
[\![\neg\phi]\!] &= \overline{[\![\phi]\!]}
\end{aligned}
$$

ULB

Symbolic Computation (cont'd)

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Existential Modalities

- symbolic computation for $\mathsf{EX}\phi$ using $[\![\mathsf{EX}\phi]\!] = \bigcup_{i\in[1,k]}[\![\mathsf{EX}[i]\phi]\!]$:

$$\mathcal{I}_{EX\phi} = \bigcup_{i\in[1,k]}(\mathcal{I}_{\phi}^{[x_i\leftarrow(x_i-1)]} \cap \mathcal{I}_{\top})$$

- for $\mathsf{EG}\phi$ and $\mathsf{E}[\phi_1 \mathsf{U} \phi_2]$, use classical fixed point:

$$\begin{array}{rcl} [\![\mathsf{EG}\phi]\!] & = & \textbf{gfp } \lambda\psi \cdot [\![\phi]\!] \cap [\![\mathsf{EX}\psi]\!] \\ [\![\mathsf{E}[\phi_1 \mathsf{U} \phi_2]]\!] & = & \textbf{lfp } \lambda\psi \cdot [\![\phi_2]\!] \cup ([\![\phi_1]\!] \cap [\![\mathsf{EX}\psi]\!]) \end{array}$$

- for $\mathsf{EF}\phi \stackrel{def}{=} \mathsf{E}[\top \mathsf{U} \phi]$, we can compute $\mathcal{I}_{\mathsf{EF}\phi} = \downarrow\mathcal{I}_{\phi} \cap \mathcal{I}_{\top}$

Symbolic Computation (cont'd)

Universal Modalities

- symbolic computation for AXϕ using $[\![\mathsf{AX}\phi]\!] = [\![\neg\mathsf{EX}\neg\phi]\!]$:

$$\mathcal{I}_{AX\phi} = \overline{\mathcal{I}_{\mathsf{EX}\neg\phi}}$$

- for AGϕ and A$[\phi_1 \mathsf{U} \phi_2]$, use classical fixed point:

$$\begin{array}{rcl} [\![\mathsf{AG}\phi]\!] & = & \mathbf{gfp}\ \lambda\psi \cdot [\![\phi]\!] \cap [\![\mathsf{AX}\psi]\!] \\ [\![\mathsf{A}[\phi_1 \mathsf{U} \phi_2]]\!] & = & \mathbf{lfp}\ \lambda\psi \cdot [\![\phi_2]\!] \cup ([\![\phi_1]\!] \cap [\![\mathsf{AX}\psi]\!]) \end{array}$$

- for AGϕ, since $[\![\mathsf{AG}\phi]\!] = [\![\neg\mathsf{EF}\neg\phi]\!]$, we can compute $\mathcal{I}_{\mathsf{AG}\phi} = \overline{\downarrow\mathcal{I}_{\neg\phi} \cap \mathcal{I}_{\top}}$

Experimental Results

Model	#proc	#events	IST (in sec.)	NuSMV (in sec.)
Peterson	2	2000	0.46	349.57
	2	5000	7.53	↑↑
	2	15000	189.65	↑↑
Peterson Generalized	2	2000	0.20	294.46
	2	5000	6.44	↑↑
	2	20000	390.90	↑↑
	5	1000	2.04	13.74
	5	1500	6.82	↑↑
	5	5000	176.62	↑↑
	10	1500	7.53	150.23
	10	2000	27.01	↑↑
	10	5000	147.89	↑↑

↑↑ indicates ($>$ 10 min.)

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Experimental Results

Model	#proc	#events	IST (in sec.)	NuSMV (in sec.)
Alternating Bit Protocol	2	1000	13.60	297.28
	2	2000	27.56	↑↑
	2	5000	257.29	↑↑
Dining Philosophers	3	100	0.15	6.36
	3	200	1.11	↑↑
	3	2000	366.22	↑↑
	5	100	0.25	↑↑
	5	200	27.05	↑↑
	5	500	125.56	↑↑
	10	100	1.67	↑↑
	10	200	26.94	↑↑
	10	500	↑↑	↑↑

↑↑ indicates ($>$ 10 min.)

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Conclusion

What has been done?

- we proposed a symbolic technique for testing full CTL properties on distributed systems
- has been implemented and works well in practice

Future Work

- integration of the methods in our tool TraX
- interface with our industrial design environment $_{\mathrm{d}}$SL [DMM03,DGMM05]
- investigate possible further improvements of our technique
- investigate similar models (MSC...)
- comparison with other symbolic data structures (IDD,...)

Conclusion

Questions?