From Two-Way to One-Way Finite State Transducers

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Finite State Automata

- finite string acceptors over a finite alphabet Σ
- read-only input tape, left-to-right
- finite set of states

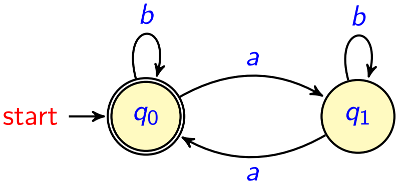
Definition (Finite State Automaton)

A finite state automaton (FA) on Σ is a tuple $A = (Q, I, F, \delta)$ where

- Q is the set of states,
- $I \subseteq Q$, reps. $F \subseteq Q$ is the set of initial, resp. final, states,
- $\delta : Q \times \Sigma \to Q$ is the transition relation.

$$L(A) \;=\; \{w \in \Sigma^* \mid \text{there exists an accepting run on } w\}$$

Finite State Automata – Example



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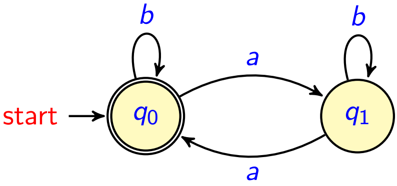
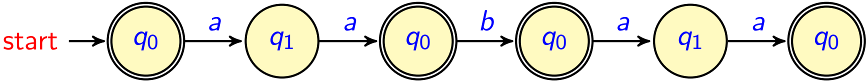


Run on *aabaa*:



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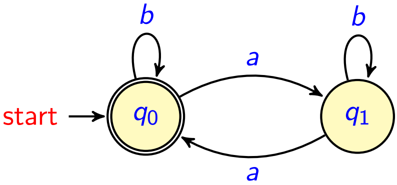

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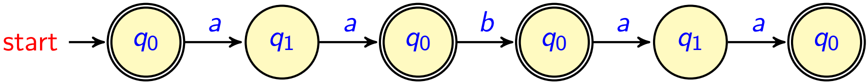

$$L(A) = \{w \in \Sigma^* \mid w \text{ contains an even number of } a\}$$

From Languages to Transductions

Let Σ and Δ be two finite alphabets.

Definition

Language on Σ	Transduction from Σ to Δ
function from Σ^* to $\{0,1\}$	relation $R \subseteq \Sigma^* \times \Delta^*$
defined by automata	defined by transducers
accept strings	transform strings

transducer $=$ <u>automaton</u> $+$ output mechanism.

One-Way Finite State Transducers

Finite State Transducers

- read-only left-to-right input head
- write-only left-to-right output head
- finite set of states

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Definition (Finite State Transducers)

A finite state transducer from Σ to Δ is a pair $T = (A, O)$ where

- $A = (Q, I, F, \delta)$ is the underlying automaton
- O is an output morphism from δ to Δ^*.

- If $t = q \xrightarrow{a} q' \in \delta$, then $O(t)$ defines its output.
- $q \xrightarrow{a|w} q'$ denotes a transition whose output is $w \in \Delta^*$.

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Two classes of transducers:

- DFT if A is deterministic
- NFT if A is non-deterministic.

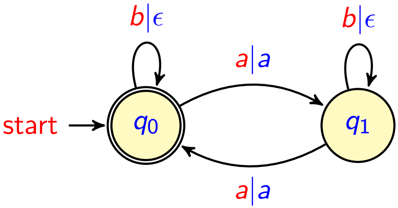
Some applications

- language and speech processing (e.g. see work by Mehryar Mohri)
- model-checking infinite state-space systems[1]
- string pattern matching
- verification of web sanitizers[2]

[1] A survey of regular model checking, P. Abdulla, B. Jonsson, M. Nilsson, M. Saksena. 2004

[2] see BEK, developped at Microsoft Research

Finite State Transducers – Example 1



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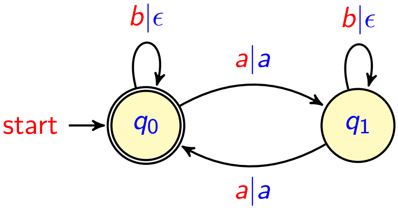

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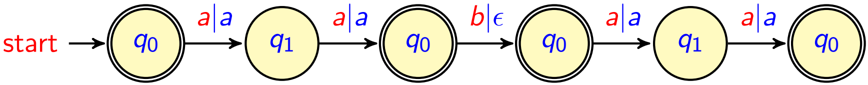

$$T(aabaa) = a.a.\epsilon.a.a = aaaa.$$

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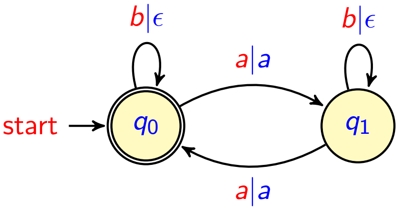

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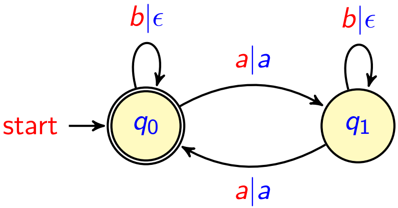

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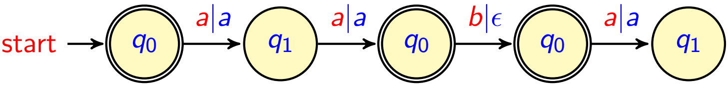

$T(aaba) =$ **undefined**

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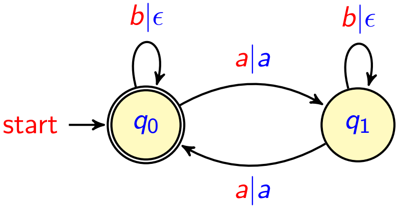
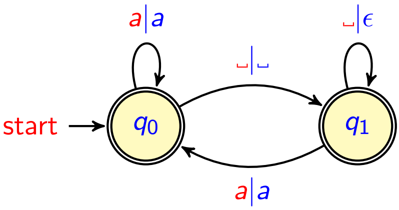


Semantics

$$
\begin{aligned}
dom(T) &= \{w \in \Sigma^* \mid \#_a w \text{ is even}\} \\
R(T) &= \{(w, a^{\#_a w}) \mid w \in dom(T)\}
\end{aligned}
$$

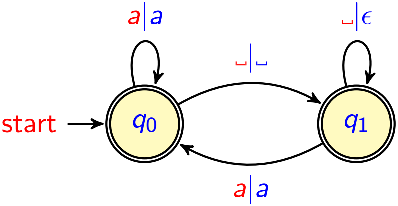
Finite State Transducers – Example 2

␣ = white space



Finite State Transducers – Example 2

$\textvisiblespace$ = white space



Semantics

Replace blocks of consecutive white spaces by a single white space.

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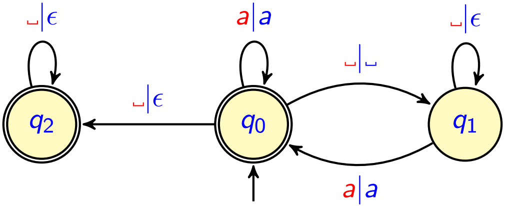
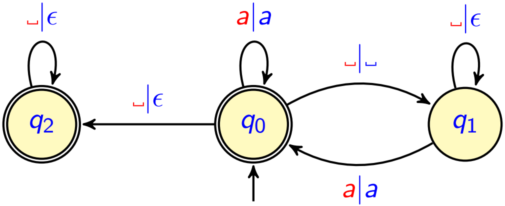


Finite State Transducers – Example 3

$\llcorner\!\lrcorner$ = white space



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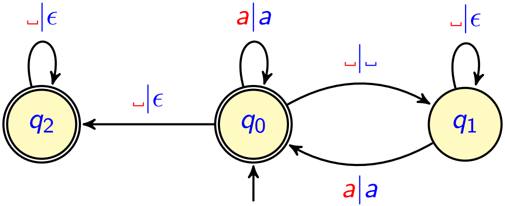

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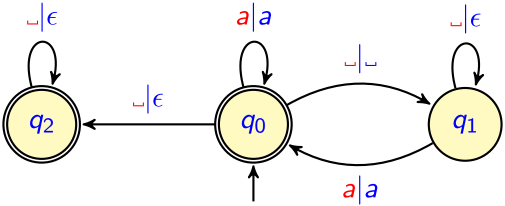
$$T(\text{␣␣}aa\text{␣␣␣}a\text{␣␣}) = \text{␣}aa\text{␣}a$$

Non-deterministic but still defines a function: functional NFT

Is non-determinism needed ?



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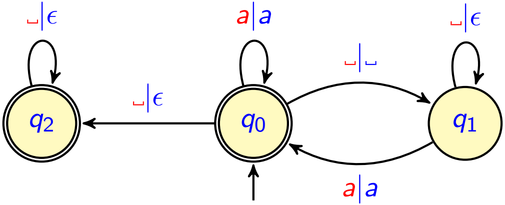

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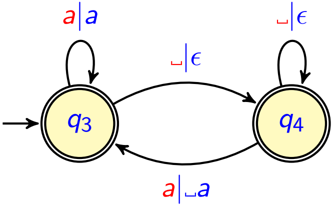
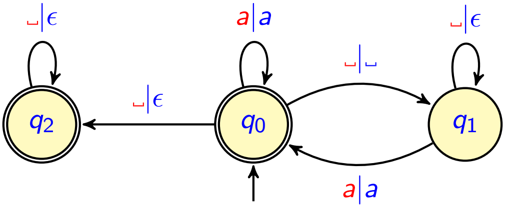


How to get a deterministic FT ?



- extend automata subset construction with outputs
- output the longest common prefix

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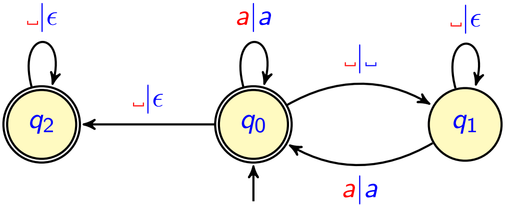

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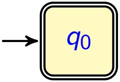

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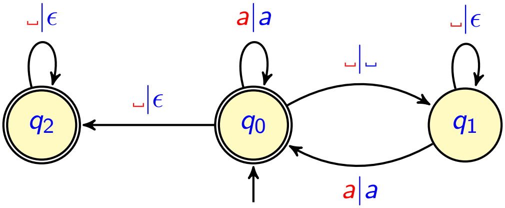

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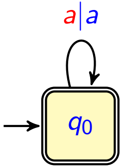

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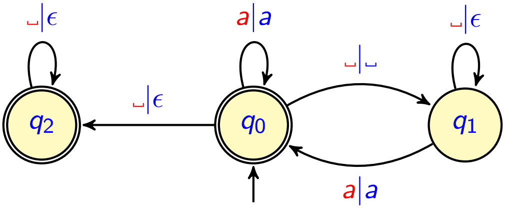

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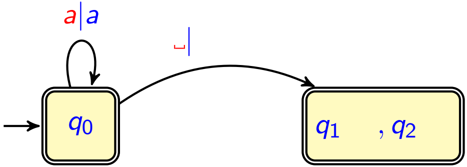

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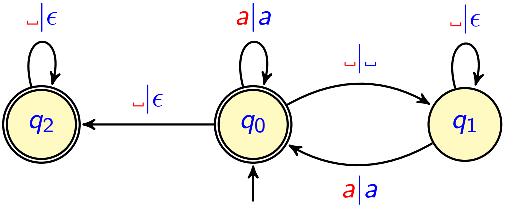

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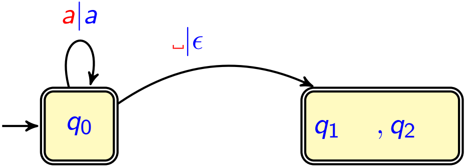

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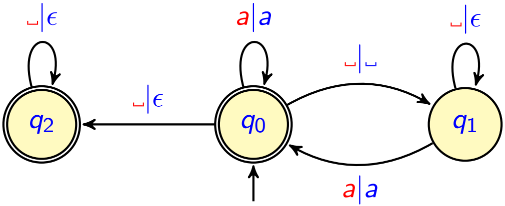

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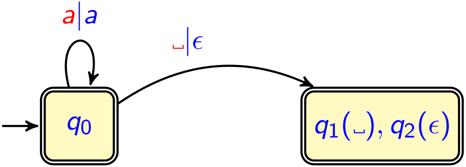

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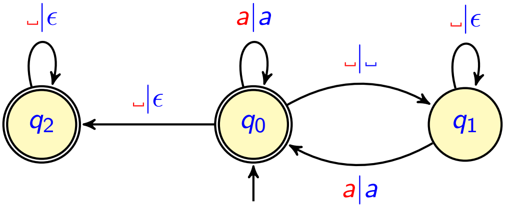

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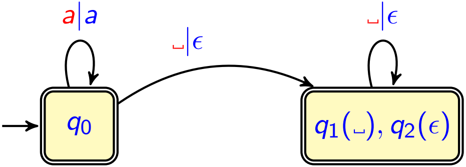

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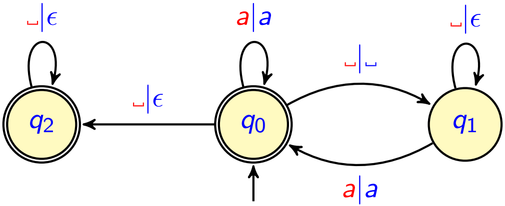

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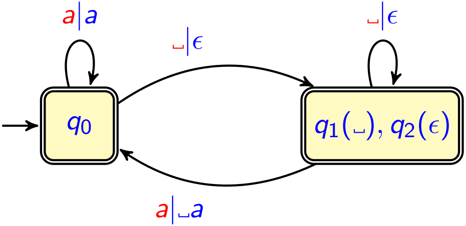

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- not in general: DFT define functions, NFT define relations
- what about functional NFT ?

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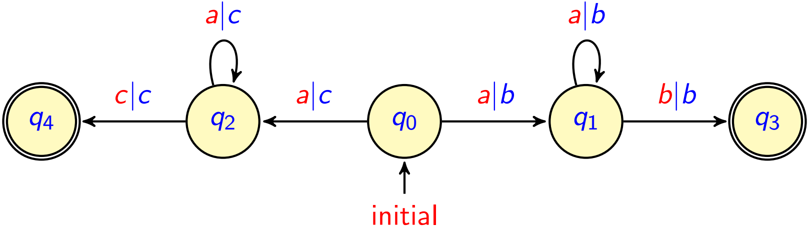
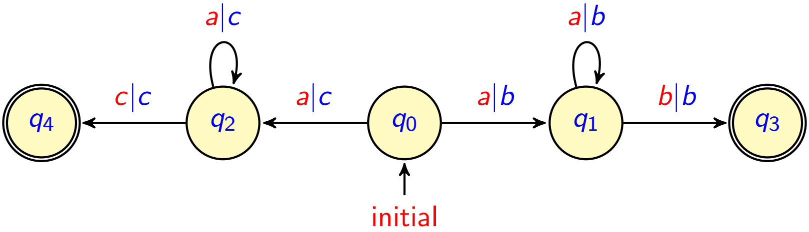


Semantics

$$R(T) : \begin{cases} a^n b \mapsto b^{n+1} \\ a^n c \mapsto c^{n+1} \end{cases}$$

functional but not determinizable

Subset construction fails ...



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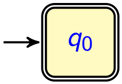

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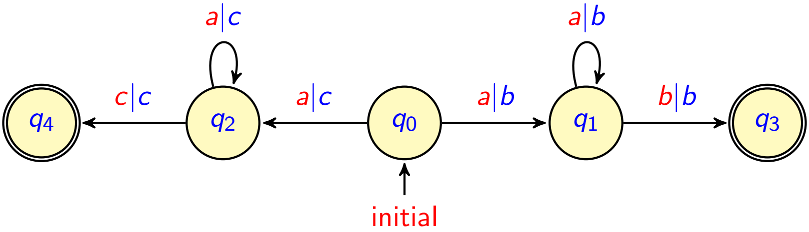

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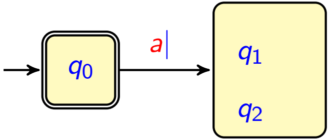

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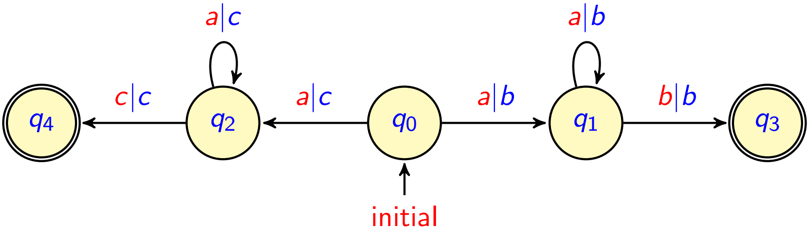

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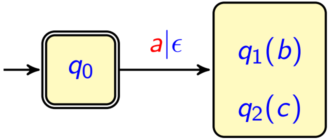

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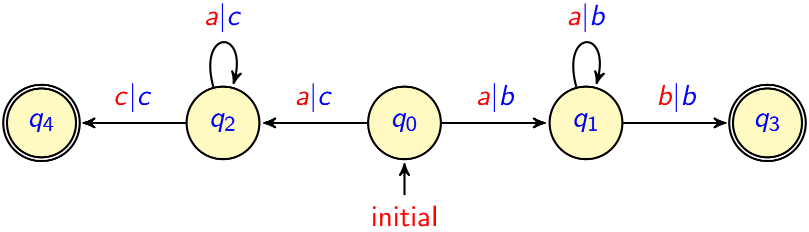

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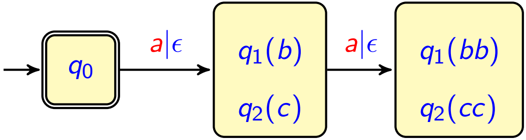

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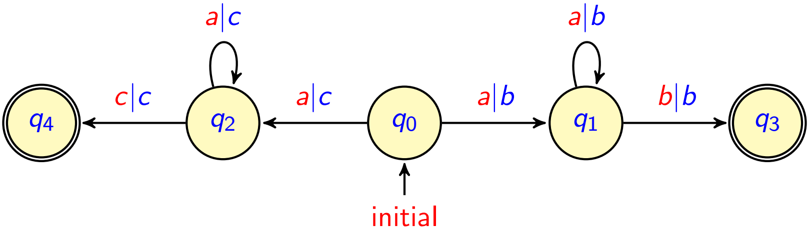

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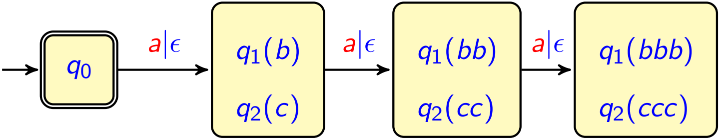

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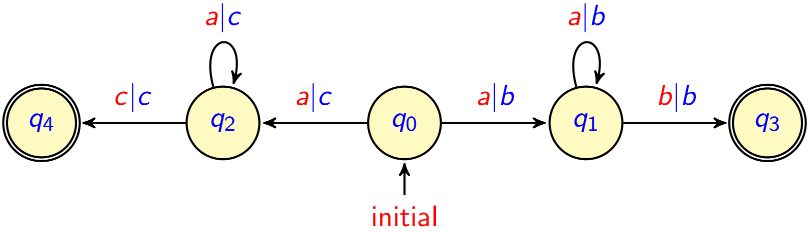

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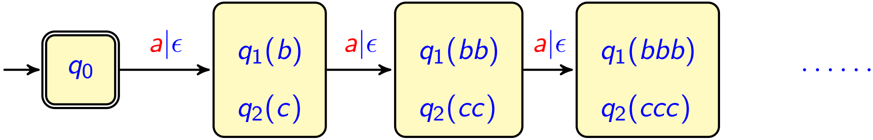

How to guarantee termination of subset construction?

LAG

$LAG(u, v) = (u', v')$ such that $u = \ell u'$, $v = \ell v'$ and $\ell = lcp(u, v)$.

E.g. $LAG(abbc, abc) = (bc, c)$.

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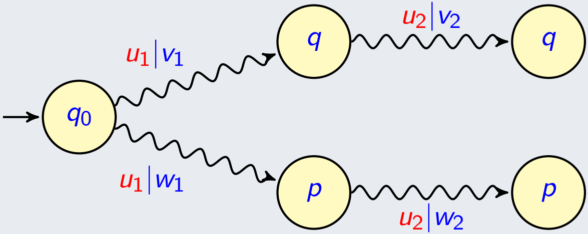
Lemma (Twinning Property)

Subset construction terminates **iff** *for all such situations*



it is the case that $LAG(v_1, w_1) = LAG(v_1v_2, w_1w_2)$.

Determinizability is decidable

Theorem (Choffrut 77, Beal Carton Prieur Sakarovitch 03)

Given a functional NFT T, the following are equivalent:

1. *it is determinizable*
2. *the twinning property holds.*

Moreover, the twinning property is decidable in PTime.

Application: analysis of streaming transformations

Bounded Memory Problem

Hypothesis:

- input string is received as a (very long) **stream**
- output string is produced as a stream

Input: a transformation defined by some functional NFT

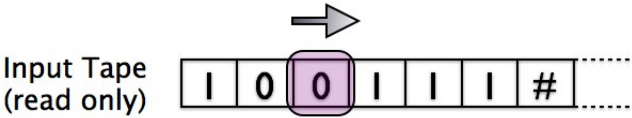
Output: can I realize this transformation with bounded memory ?

$$\exists B \in \mathbb{N} \cdot \forall u \in dom(T)$$

$T(u)$ can be computed with B-bounded memory ?

Streaming Model

Deterministic Turing Transducer



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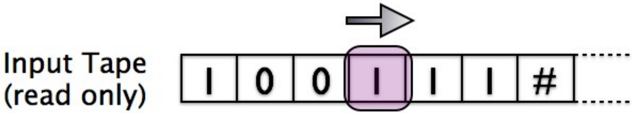

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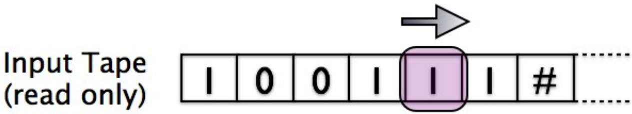

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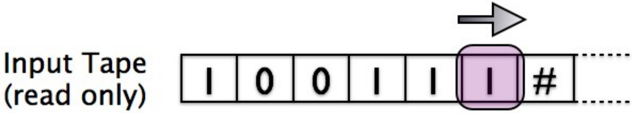

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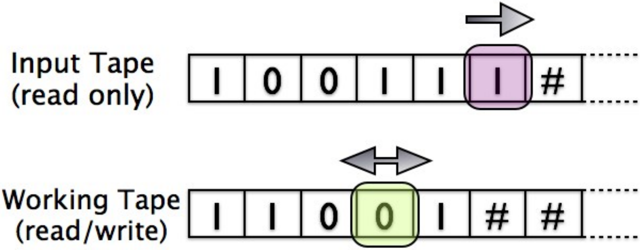

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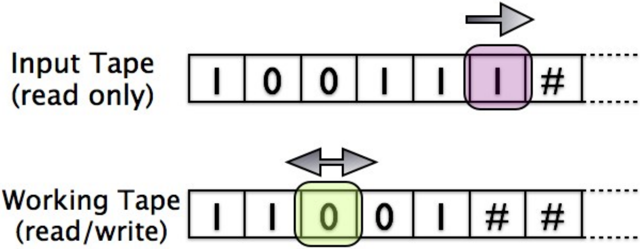

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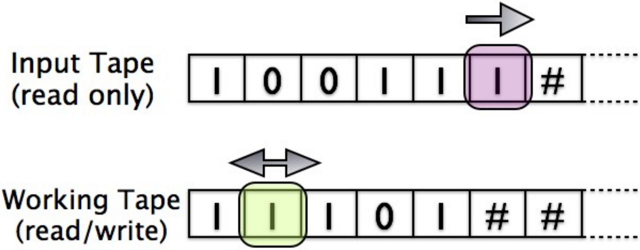

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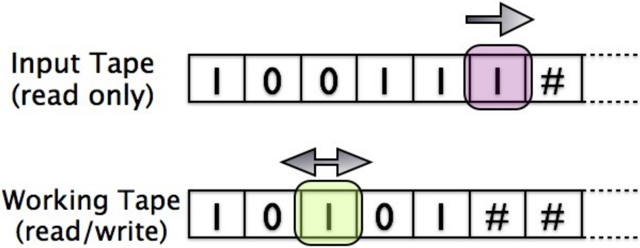

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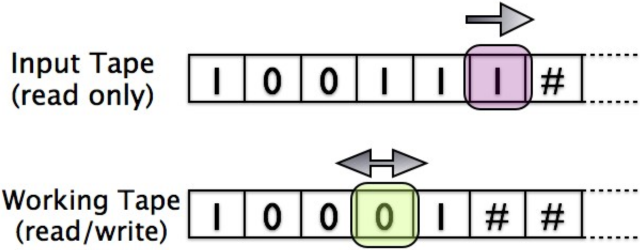

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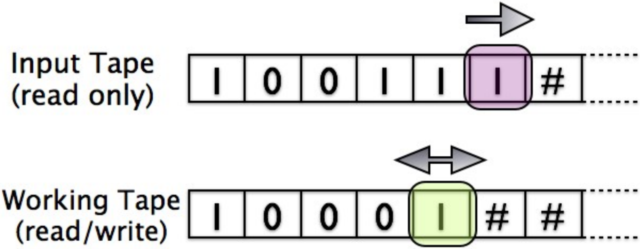

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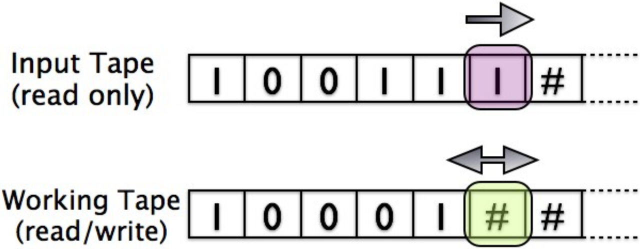

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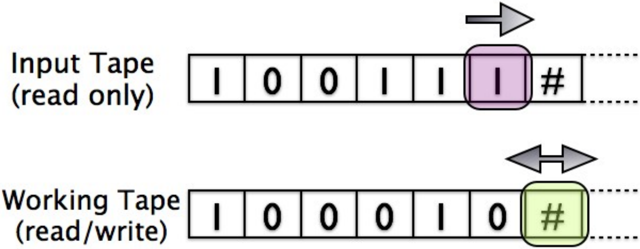

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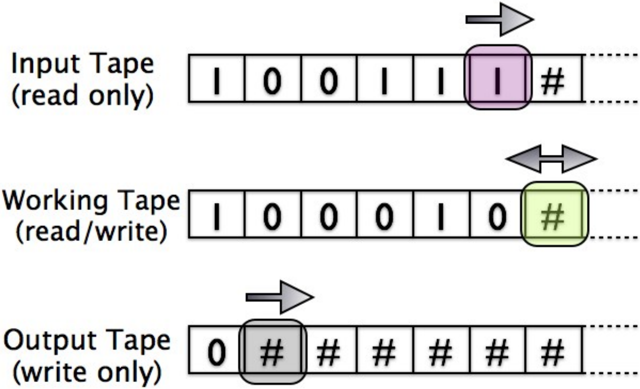

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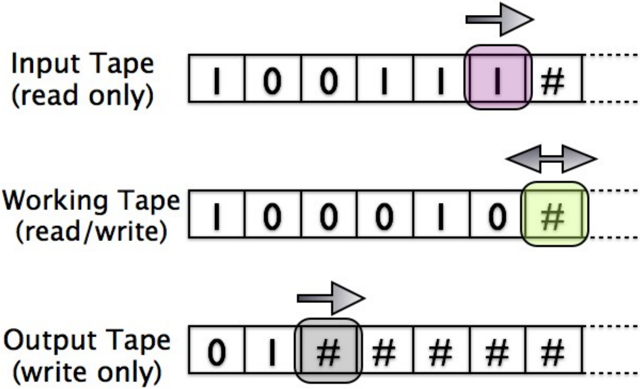

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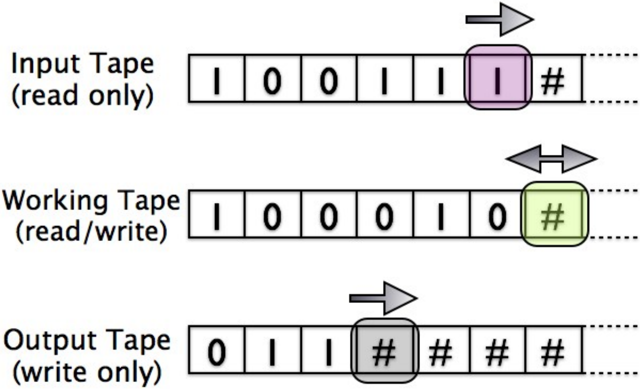

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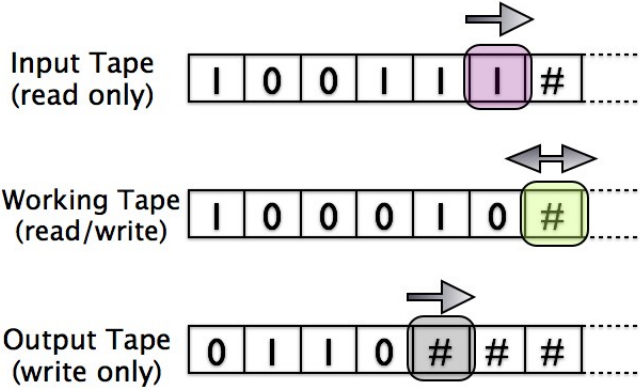

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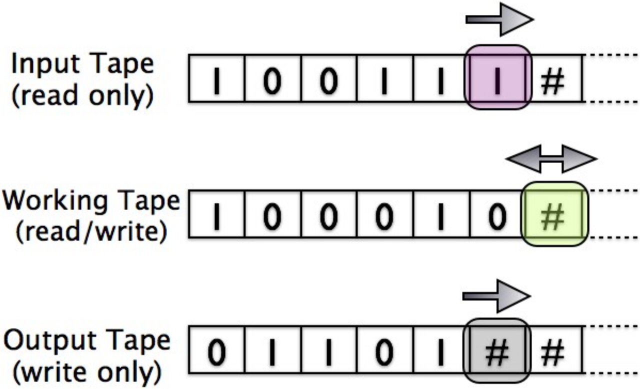

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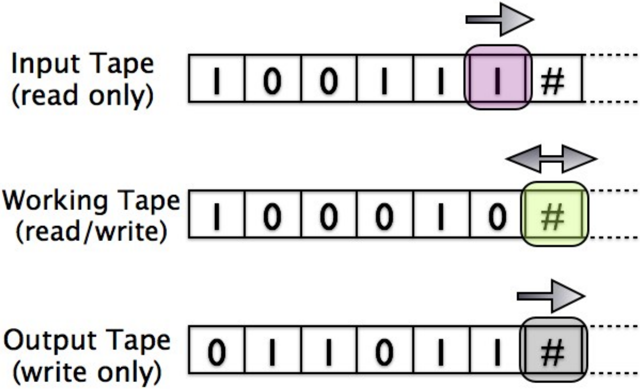

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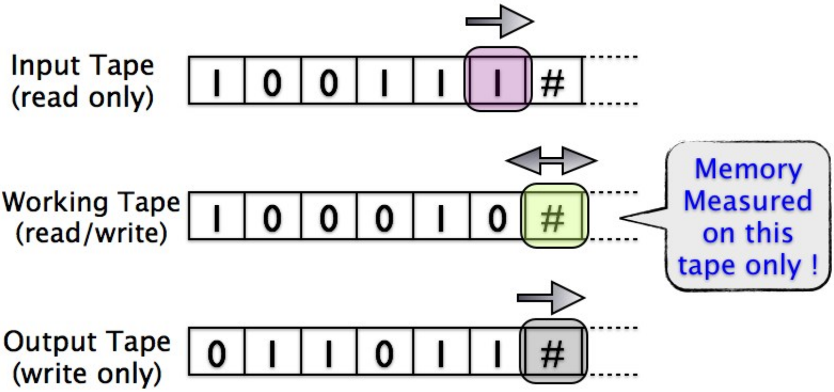

Bounded Memory Problem – Examples

$T_1 : \begin{cases} a^n b \mapsto b^{n+1} \\ a^n c \mapsto c^{n+1} \end{cases}$	Not bounded memory
$T_2 : \textvisiblespace\textvisiblespace a \textvisiblespace\textvisiblespace\textvisiblespace b \textvisiblespace\textvisiblespace \mapsto \textvisiblespace a \textvisiblespace b$	Bounded memory

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$T_2 : \sqcup\sqcup a \sqcup\sqcup\sqcup b \sqcup\sqcup \mapsto \sqcup a \sqcup b$	Bounded memory

Theorem

For all functional NFT T, the following are equivalent:

1. T *is bounded memory*
2. T *is determinizable*
3. T *satisfies the twinning property.*

Proof based on the following two observations:

1. any DFT is bounded memory
2. bounded memory Turing Transducer $\equiv$ DFT

Corollary

Corollary

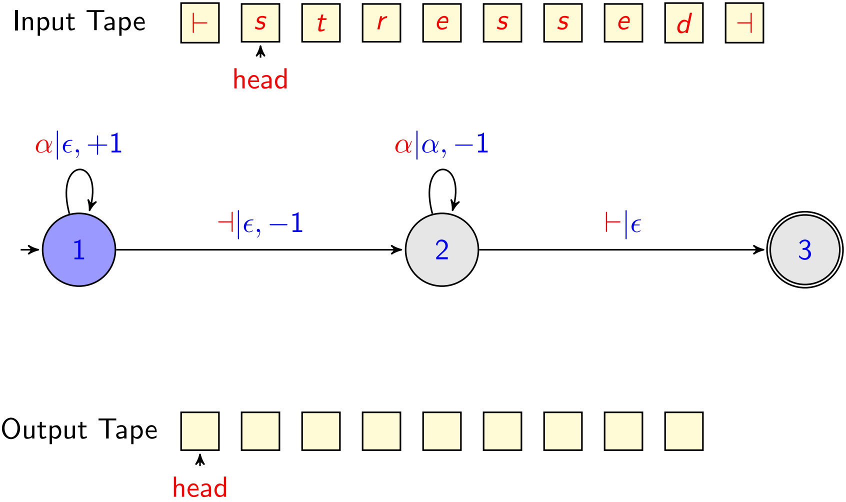
For all transductions R, the following are equivalent:

1. R *is computable with bounded memory*
2. R *is definable by some DFT*

Two-Way Finite State Transducers

extending finite state transducers with a two-way input tape.

Two-way finite state transducers (2NFT)



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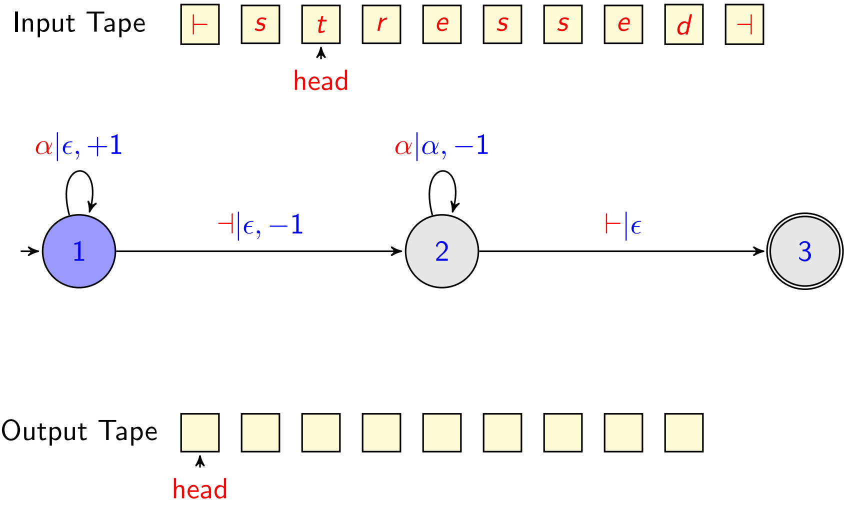

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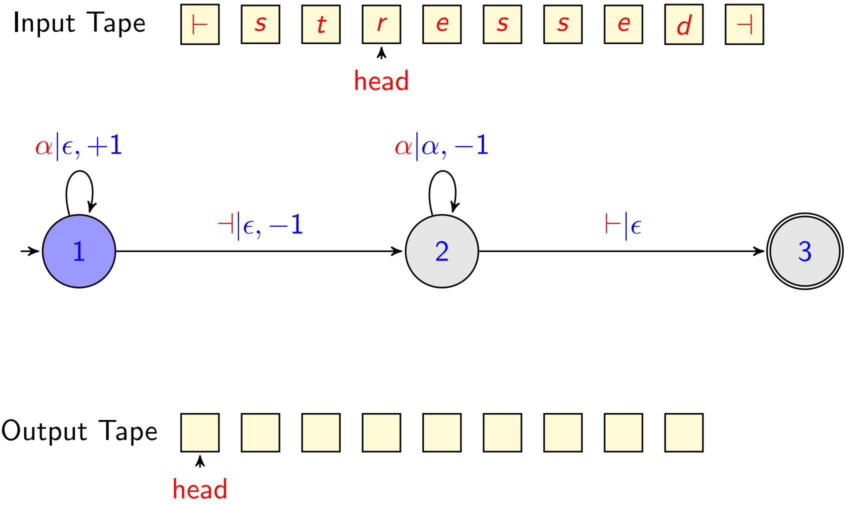

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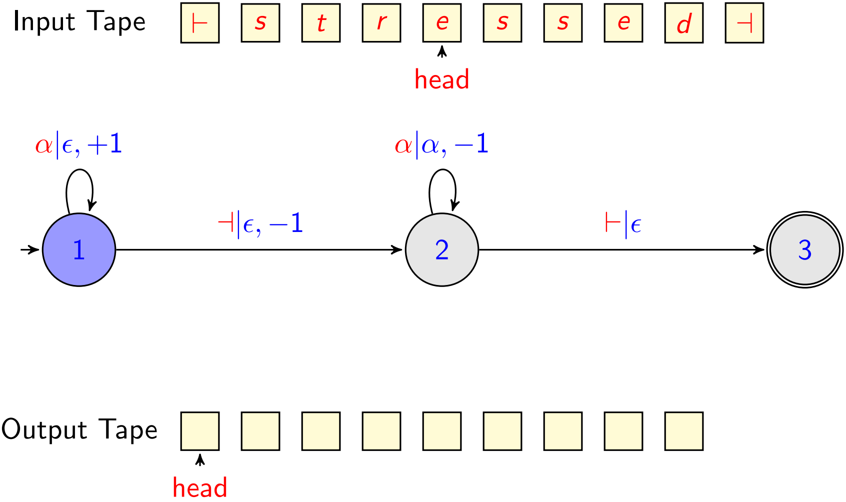

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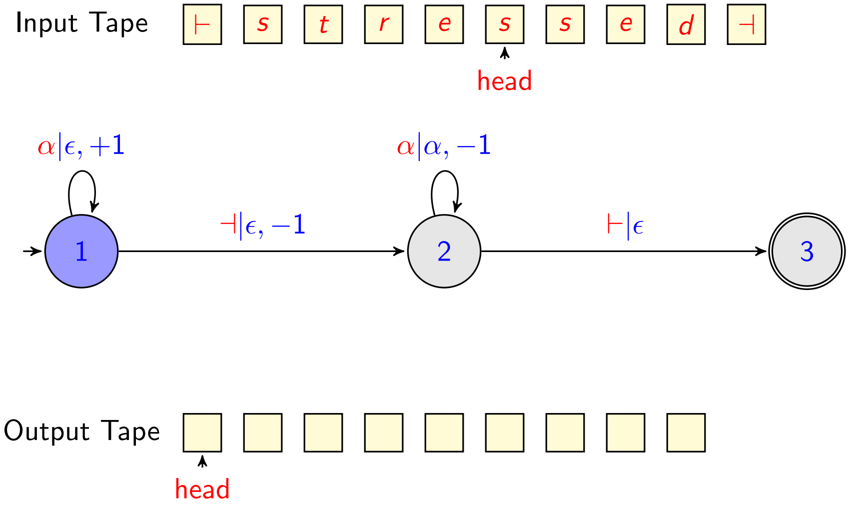

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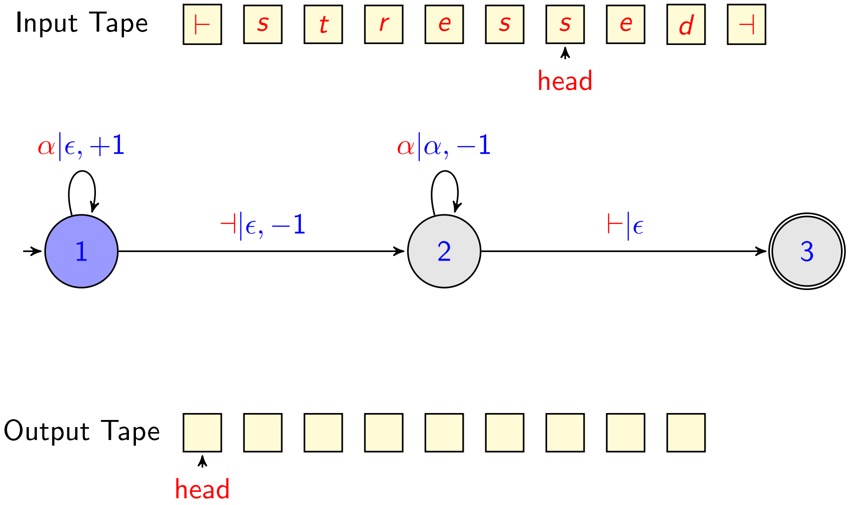

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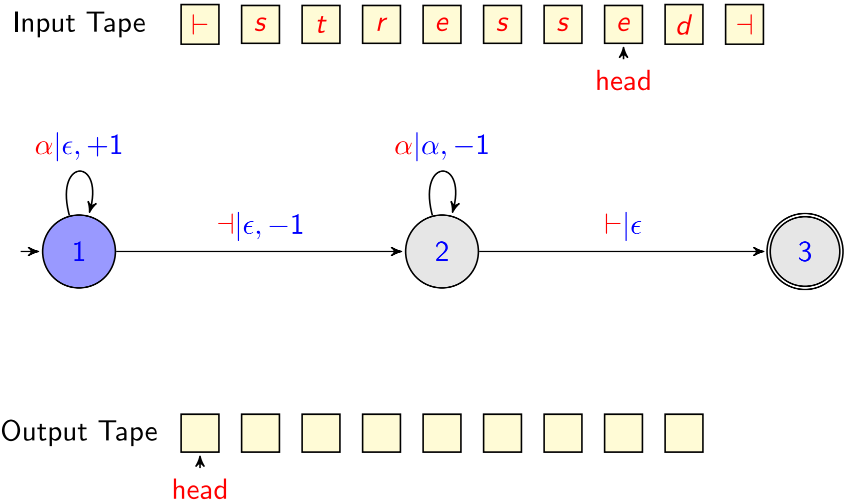

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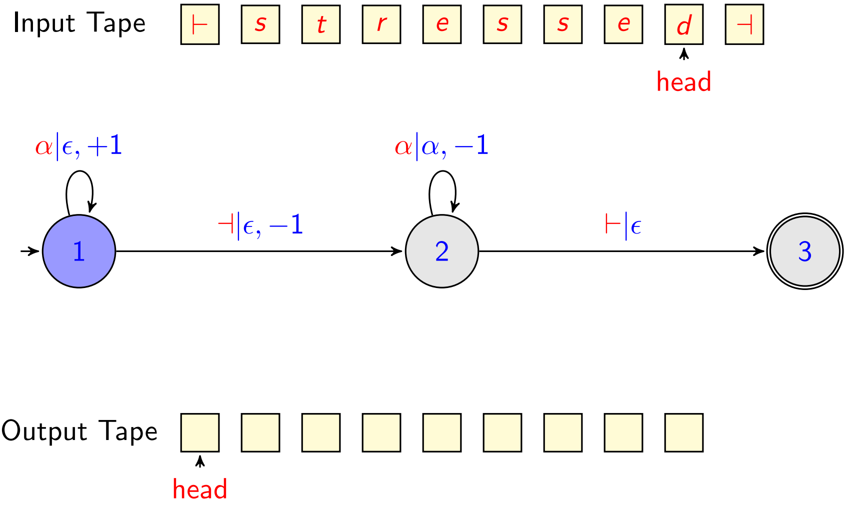

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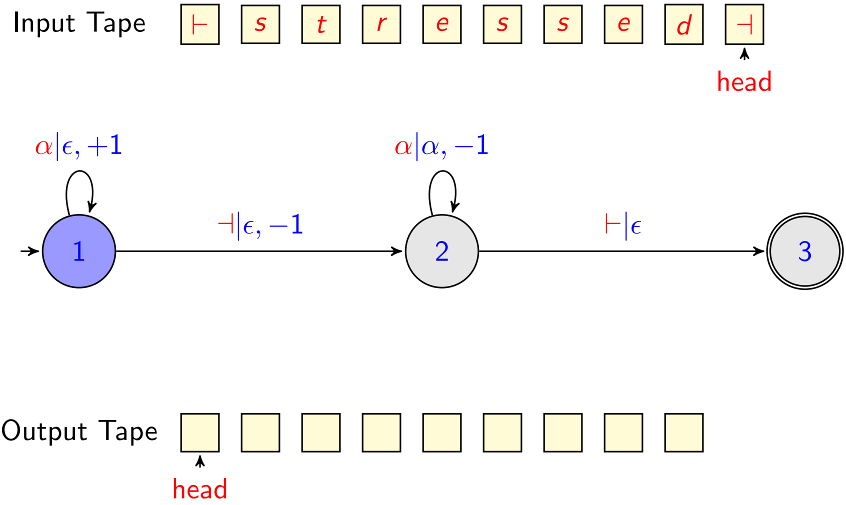

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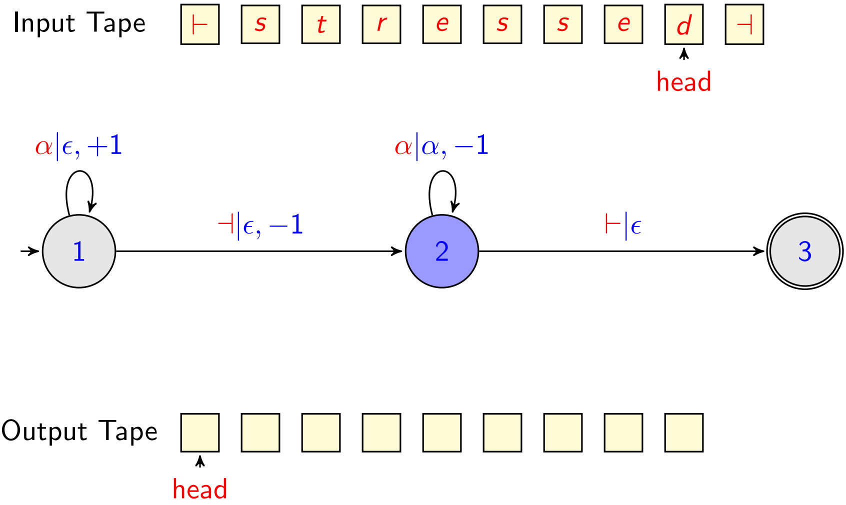

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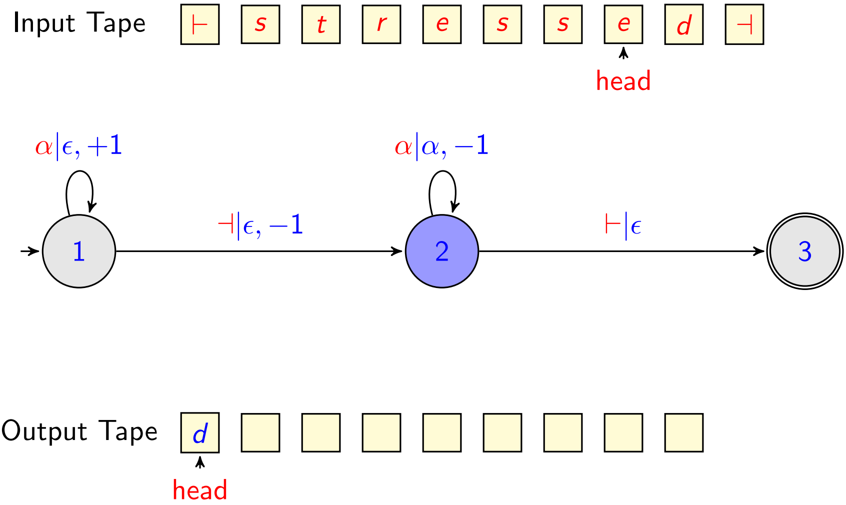

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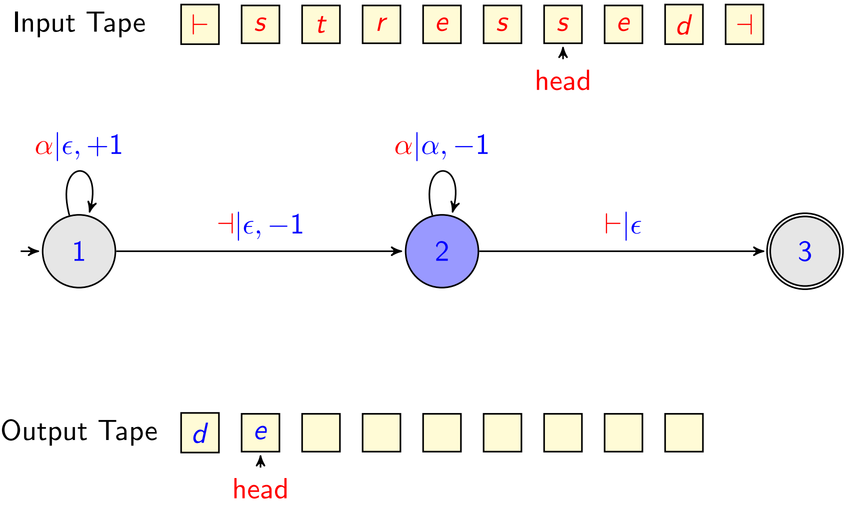

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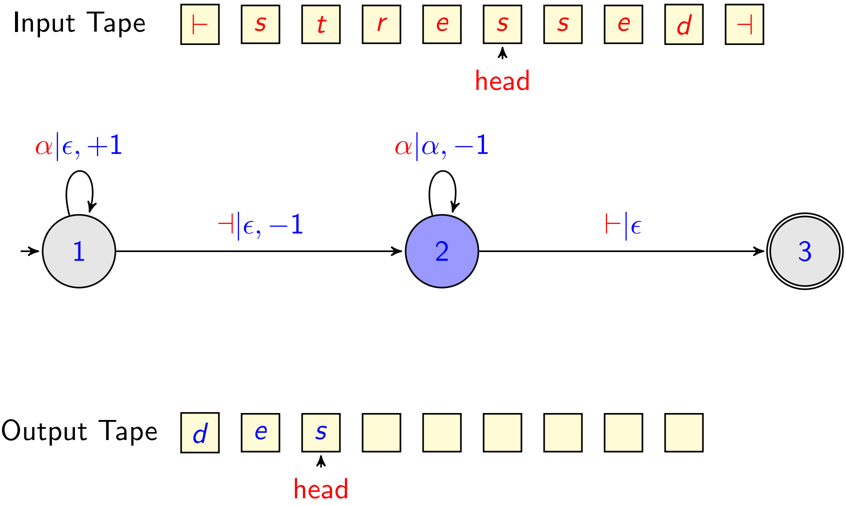

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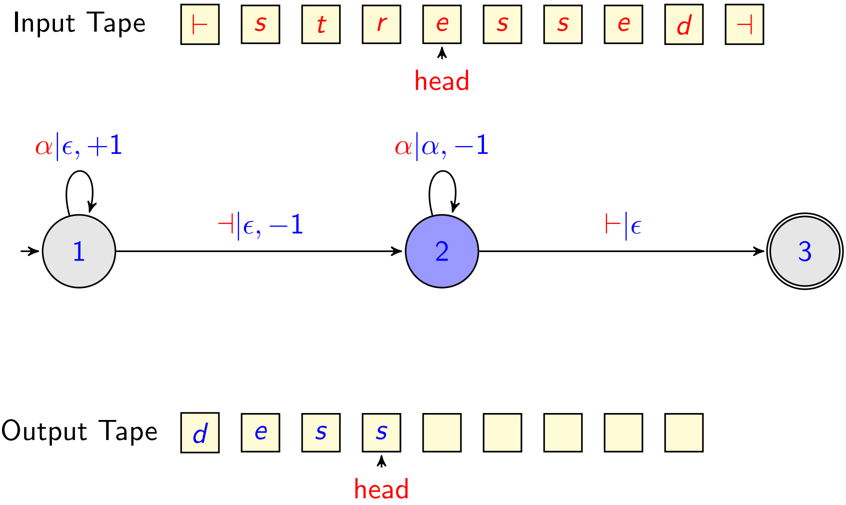

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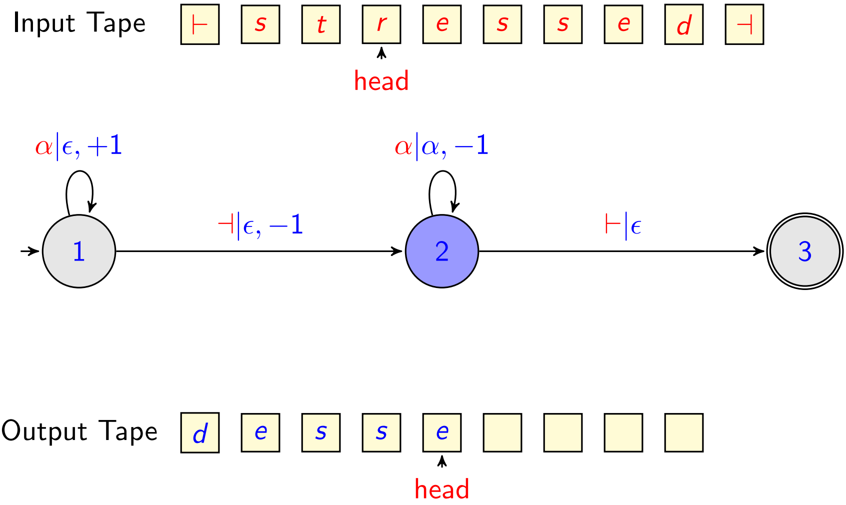

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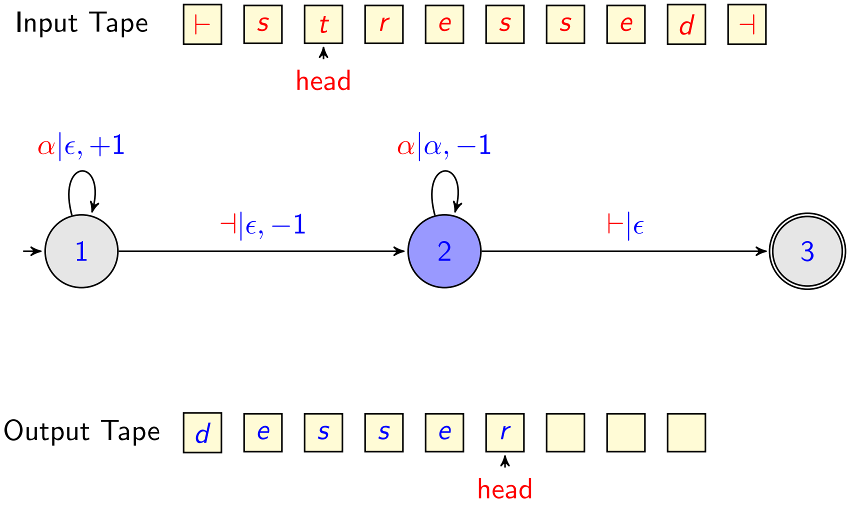

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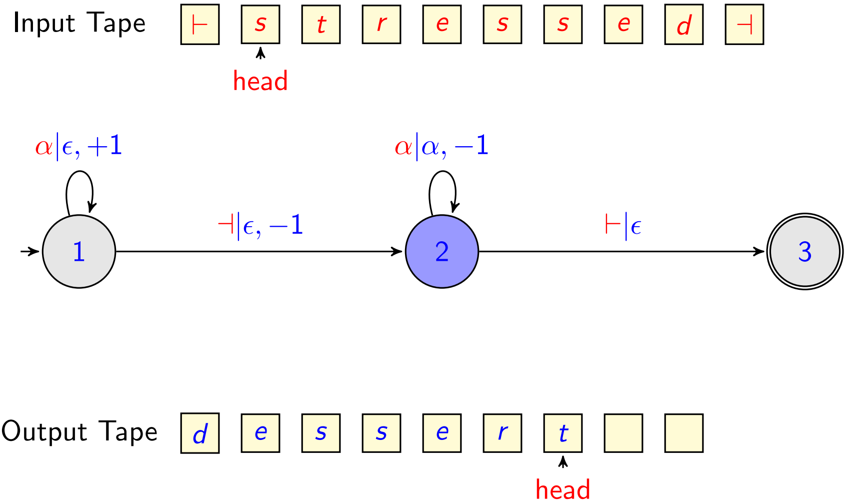

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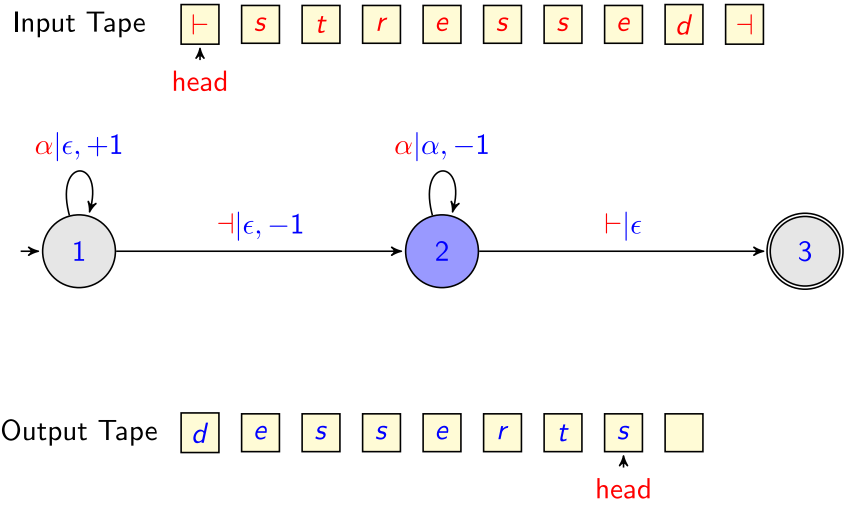

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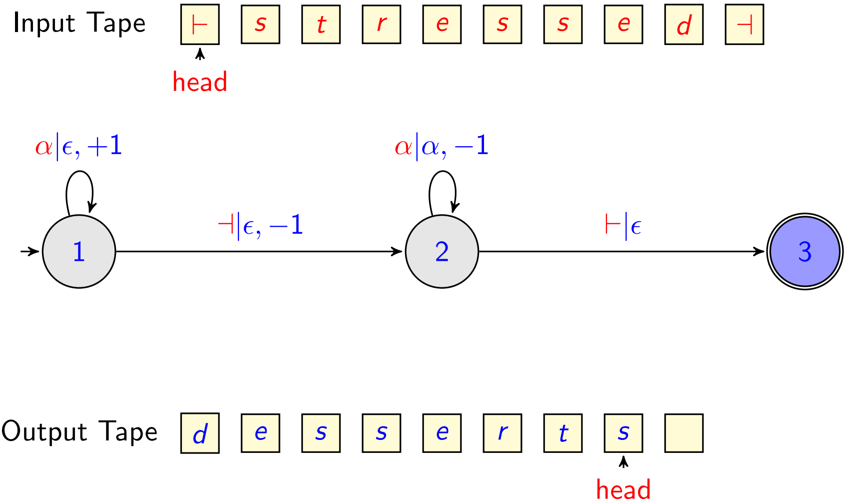

Two-way finite state transducers – Properties

Main Properties of 2NFT

1. closed under composition (Chytil Jakl 77)
2. equivalence of functional 2NFT is decidable (Culik, Karhumaki, 87)
3. functional 2NFT $\equiv$ 2DFT (Hoogeboom Engelfriet 01, De Souza 13)

Logical Characterization (Hoogeboom Engelfriet 01)

$$\text{2DFT} \equiv \text{MSO transductions}$$

2DFT define regular functions.

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2DFT define regular functions.

Also characterized by (deterministic) streaming string transducers (Alur Cerny 10).

Motivation: bounded memory problem

Question

Is the bounded memory problem decidable for 2DFT ?

Motivation: bounded memory problem

Question

Is the bounded memory problem decidable for 2DFT ?

Necessary Condition

The transduction must be definable by a one-way finite state transducer.

Indeed, bounded memory computable transductions $\equiv$ 1DFT-definable transductions

Summary

D=”(input) deterministic”

f=”functional”

DFTs	fNFTs	NFTs
2DFTs	f2NFTs	2NFTs

Summary

D="(input) deterministic"

f="functional"

$\vdash u \dashv \;\mapsto\;$ mirror(u)

DFTs	fNFTs	NFTs
$\cap$	$\cap$	$\cap$
2DFTs	f2NFTs	2NFTs

Summary

D="(input) deterministic"

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$\{(a,a),(a,b)\}$

DFTs		fNFTs	$\subsetneq$	NFTs
$\cap$		$\cap$		$\cap$
2DFTs		f2NFTs	$\subsetneq$	2NFTs

Summary

D="(input) deterministic"

f="functional"

$a^n\alpha \mapsto \alpha^{n+1}$

$$
\begin{array}{ccccc}
\text{DFTs} & \subset & \text{fNFTs} & \subset & \text{NFTs} \\
\cap & & \cap & & \cap \\
\text{2DFTs} & & \text{f2NFTs} & \subset & \text{2NFTs}
\end{array}
$$

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DFTs	$\subset$	fNFTs	$\subset$	NFTs
$\cap$		$\cap$		$\cap$
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[De Souza (13)]

$\equiv$MSOT [Engelfriet,Hoogeboom (01)]

$\equiv$ Streaming String Transducers [Alur, Černý, 2010]

Summary

D="(input) deterministic"

f="functional"

PTIME

[Choffrut (77)]

[Weber, Klemm (95)]

[Beal,Carton,Prieur,Sakarovitch(03)]

?

$$\text{DFTs} \subset \text{fNFTs} \subset \text{NFTs}$$

$$\cap \qquad\qquad \cap \qquad\qquad \cap$$

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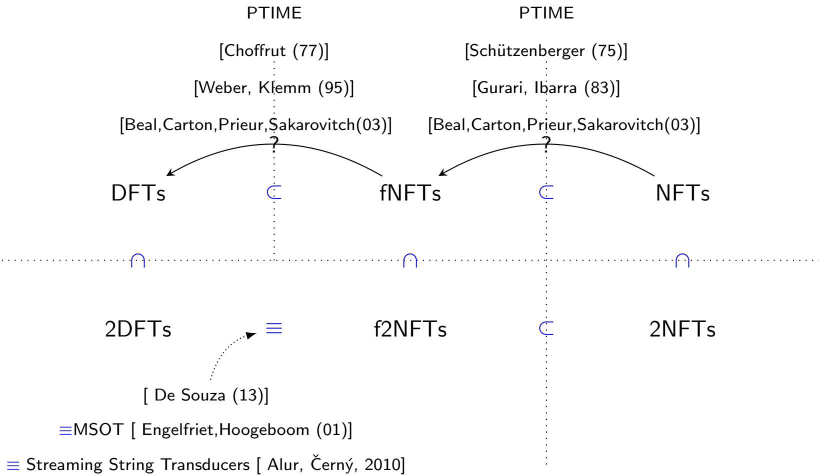

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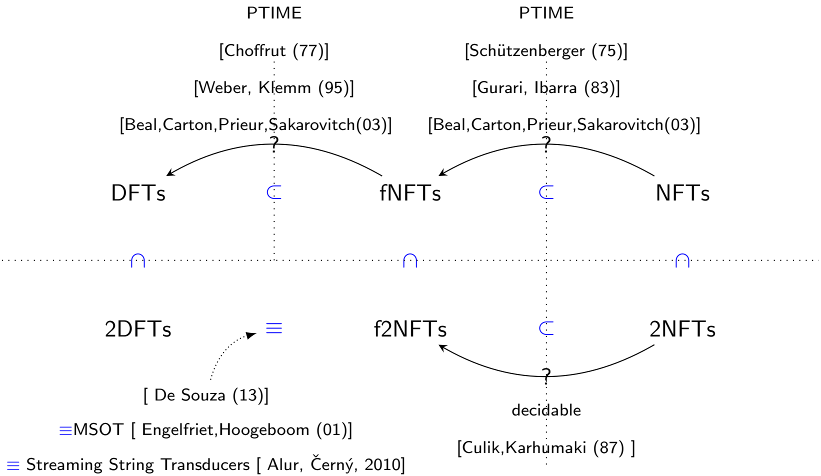

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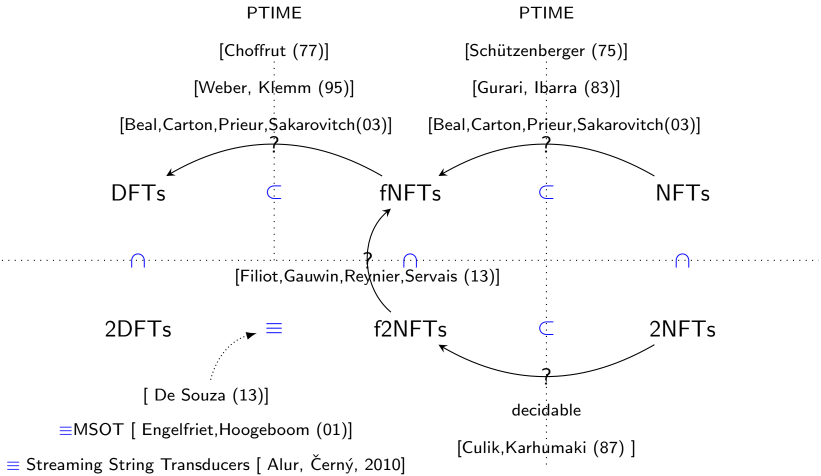

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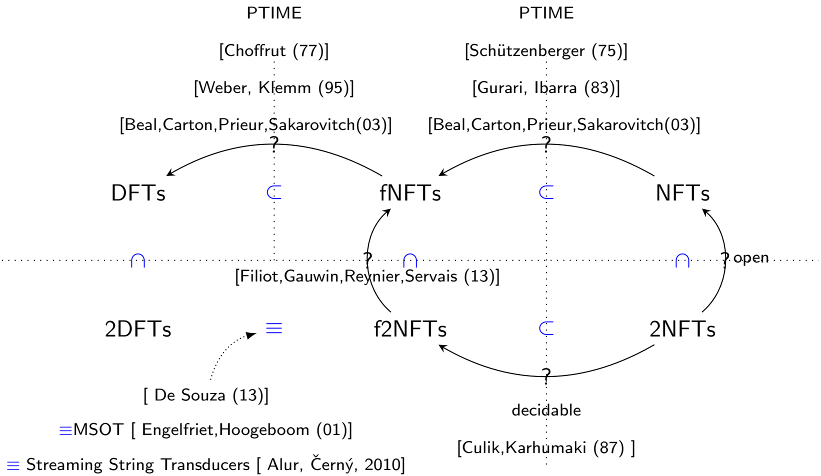


From Two-Way to One-Way Finite State Automata

1DFA vs 2DFA

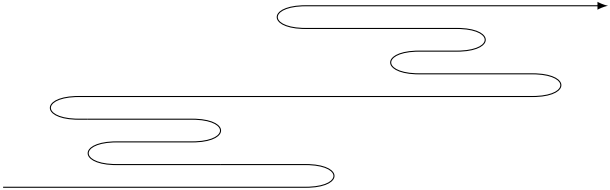
Theorem

For every 2NFA *there exists an equivalent* 1NFA.

- The first proof was done by Rabin and Scott (1959).
- In the same journal Shepherdson (1959) also published a (simpler) proof. Also rephrased, in an even simpler way, by Ullman.
- Vardi (1981) presented a different proof.
- The R&S proof is more easily adapted to transducers.

Rabin and Scott's proof for 2-Automata

- a run is made of many zigzags (moves of the input head)



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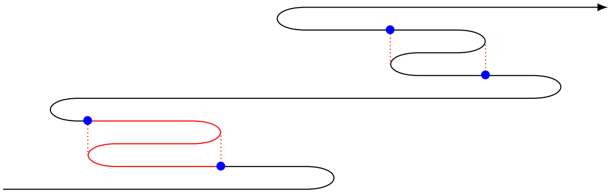
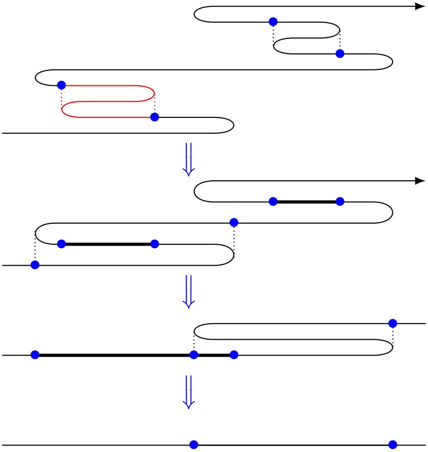


- A z-motion is an elementary zigzag.

Rabin and Scott's Proof: z-motions removal



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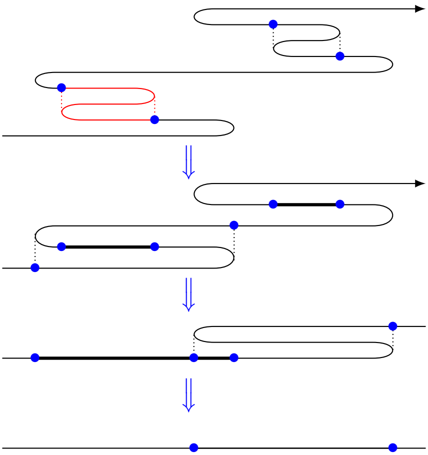
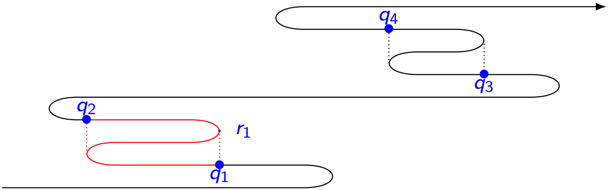


- **Def**: A shape is k crossing if any position is visited at most k times.
- **Thm**: Any k-crossing shape can be reduced to a line in k^2 steps.
- **Prop**: $\forall w \in L(A)$, w is accepted by a $|Q|$ crossing run.

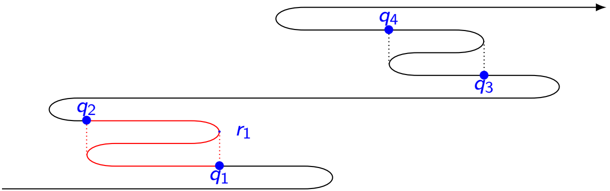
squeeze(A)



$S = \mathrm{squeeze}(A)$ removes some z-motions of A.

1. simulates A
2. non-deterministically guesses that a z-motion starts (e.g. from q_1 to q_2)
3. **checks** that is indeed a z-motion and **simulates** it one-way
4. goes back to mode 1

squeeze(A)

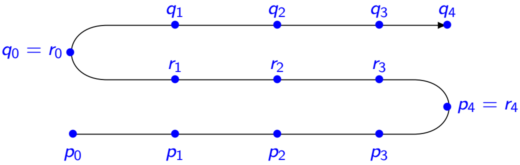


$S = \text{squeeze}(A)$ removes some z-motions of A.

Iterate squeeze(A)

- Every accepted word has a one-way run in $\text{squeeze}^{|Q|^2}(A)$ $\Longrightarrow$ remove backward transitions to obtain a 1NFA equivalent to A.

How to simulate a z-motion run in one-way ?



Simulate the three passes in parallel ! (with triple of states)

Extension to transducers

- same canvas (Rabin and Scott)
- removal of z-motion:
  ➜ translate a z-motion transducer into a fNFT
- not always possible $\rightarrow$ decision procedure

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Remarks:

- if local z-motion transductions are 1-way definable, then $\mathrm{squeeze}(T)$ can be defined
- iterate $\mathrm{squeeze}(T)$ $|Q|^2$ times (if possible), you get an equivalent 1-NFT

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Results:

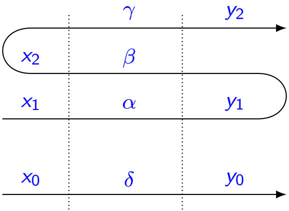
- decisition procedure to test whether a z-motion-transducer is 1-way definable
- the algorithm is complete

Decision procedure

Let T be a f2NFT.

1. repeat $|Q|^2$ times:
   - are all z-motion transductions of T NFT-definable?
     - yes: $T \leftarrow \mathrm{squeeze}(T)$
     - no: STOP: the initial 2NFTwas not NFT-definable!
2. remove backward transitions: you get an equivalent NFT

Towards a characterization of 1-way definable z-motion-transductions



$$x_1 \cdot \alpha^n \cdot y_1 \cdot \beta^n \cdot x_2 \cdot \gamma^n \cdot y_2 = x_0 \cdot \delta^n \cdot y_0$$

Lemma (Fine and Wilf (56))

Let $u, v \in \Sigma^*$. If u^ω and v^ω have a sufficiently large common factor, then $u \in (w_1 w_2)^*$ and $v \in (w_2 w_1)^*$ for some $w_1, w_2 \in \Sigma^*$.

$\Longrightarrow$ α,β,γ,δ have conjugate primitive roots (if $\neq \epsilon$).

$\rightarrow$ case analysis, depending on the emptiness of α, β, γ

Conclusion

Theorem

1. *It is decidable whether a 2DFT is definable by a 1NFT.*
2. *It is decidable whether a 2DFT is definable by a 1DFT.*
3. *Bounded memory is decidable for regular string transductions.*

- Complexity: non-elementary upper-bound, PSpace-hard.

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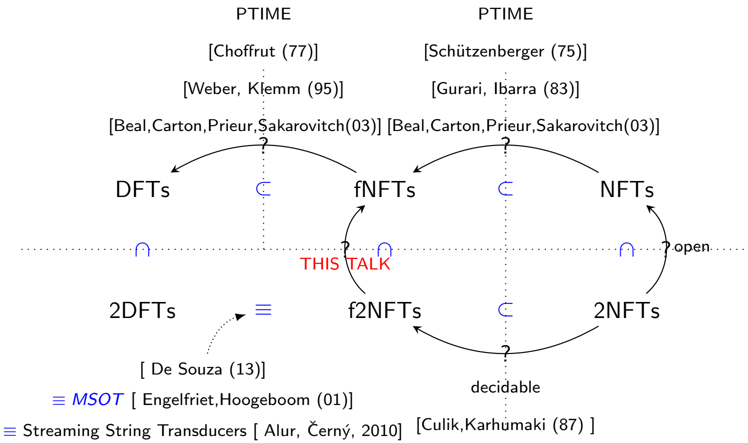
Future Work

- Lower complexity (Shepherdson)
- What about 2NFT(even non functional) ?
- Consider other structures: infinite strings, trees
- Variable minimization in streaming string transducers

Classes of Transductions

D="(input) deterministic"

f="functional"



MSO Transductions (Courcelle)

- input string seen as the logical structure over $\{succ, (lab_a)_{a \in \Sigma}\}$
- output predicates defined with MSO formulas interpreted over the input structure

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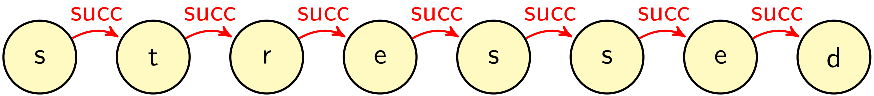

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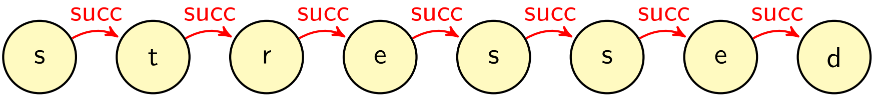

$$
\begin{aligned}
\phi_{succ}(x, y) &\equiv succ(y, x) \\
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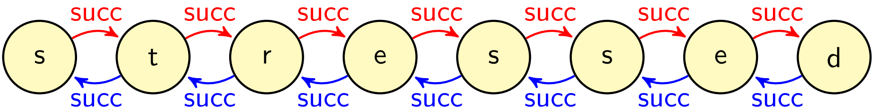

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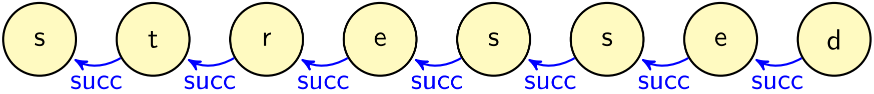

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Streaming String Transducers (Alur, Cerny, 2010)

On every transitions, a finite set of variables can be updated by

- appending a string: $x := x.u$
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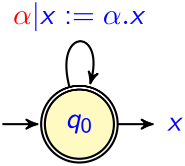

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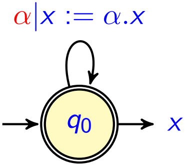
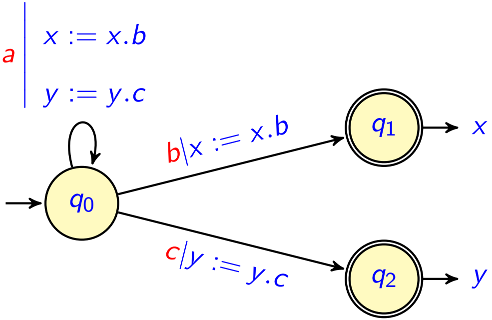




$R(T) = mirror$

$R(T) = a^n\alpha \mapsto \alpha^{n+1}$

Streaming String Transducers

Theorem (Alur Cerny 2010)

The following models are expressively equivalent:

1. *two-way DFT*
2. *MSO transductions*
3. *deterministic (one-way) streaming string transducers with copyless update*

Moreover, SSTs have good algorithmic properties and have been used to analyse list processing programs (Alur Cerny 2011).

A word about infinite strings

- most transducer models can be extended to (right-) infinite strings
- Büchi / Muller accepting conditions
- most of the results seen so far **still hold with some complications ...**

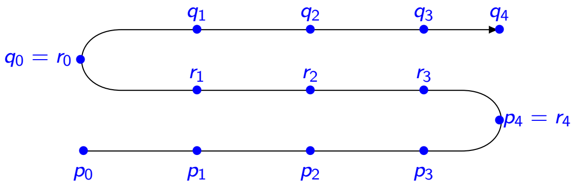
- determinization of one-way transducers: TP is too strong

- deterministic 2way $<$ functional 2way:

$$T \; : \; u \mapsto \begin{cases} a^\omega \text{ if infinite number of 'a'} \\ u \text{ otherwise} \end{cases}$$

- functional 2way $\equiv$ determinitic 2way + ω-regular look-ahead $\equiv$ ω-MSO transductions $\equiv$ ω-SST (Alur,Filiot,Trivedi,12)

z-motions simulation



It is possible to simulate a z-motion run with a one-way automaton

1. each state is a triple (p, r, q)
2. the initial state is (p_0, r_0, q_0) with $q_0 = r_0$
3. $(p_i, r_i, q_i) \xrightarrow{a} (p_{i+1}, r_{i+1}, q_{i+1})$ iff
   - $p_i \xrightarrow{a,+1} p_{i+1}$
   - $r_{i+1} \xrightarrow{a,-1} r_i$
   - $q_i \xrightarrow{a,+1} q_{i+1}$
   - final states: (p, p, q)