An Antichain Algorithm for LTL Realizability

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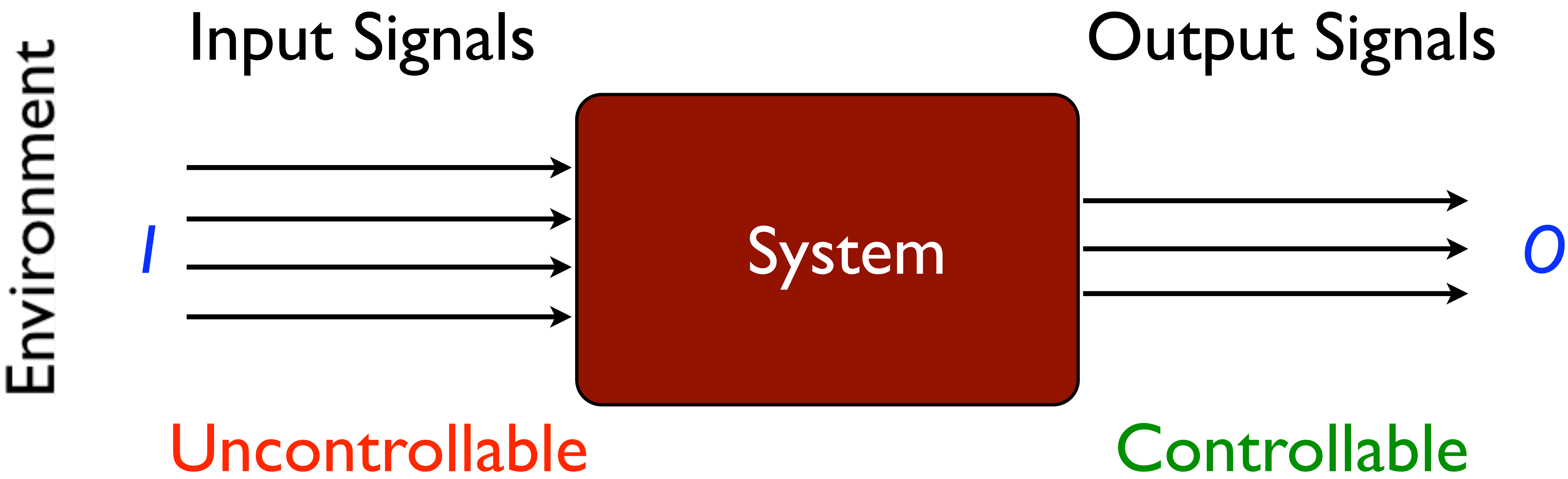
GAMES 2009, Udine

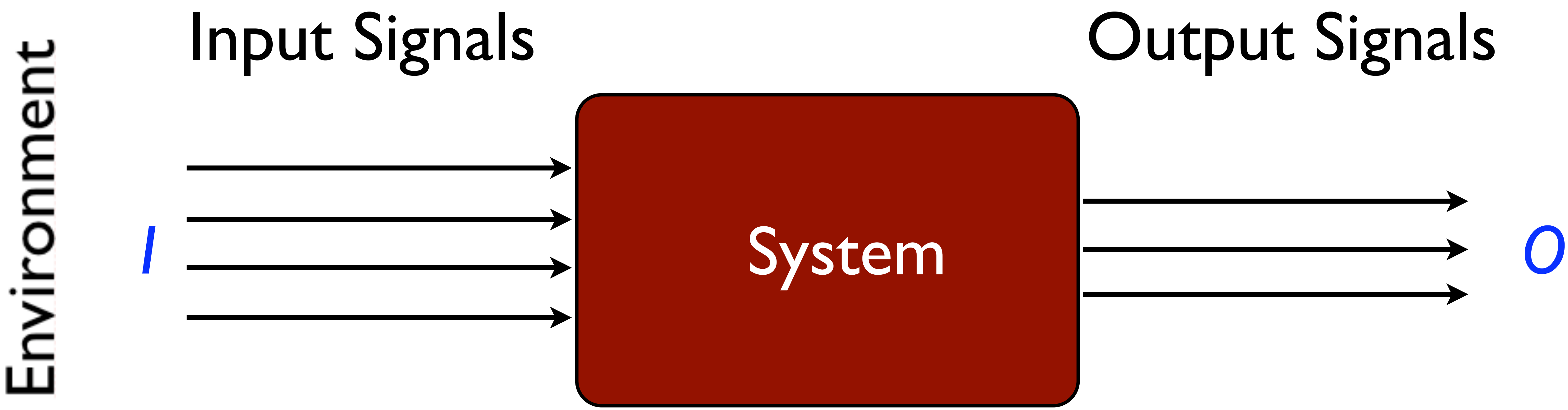
LTL Realizability



synchronous execution= infinite words over $\Sigma = 2^{I \cup O}$

$(o_0 \cup i_0)(o_1 \cup i_1)(o_2 \cup i_2)\ldots \qquad o_j \subseteq O \quad i_j \subseteq I$

LTL Realizability



Uncontrollable   Controllable

synchronous execution= infinite words over $\Sigma = 2^{I \cup O}$

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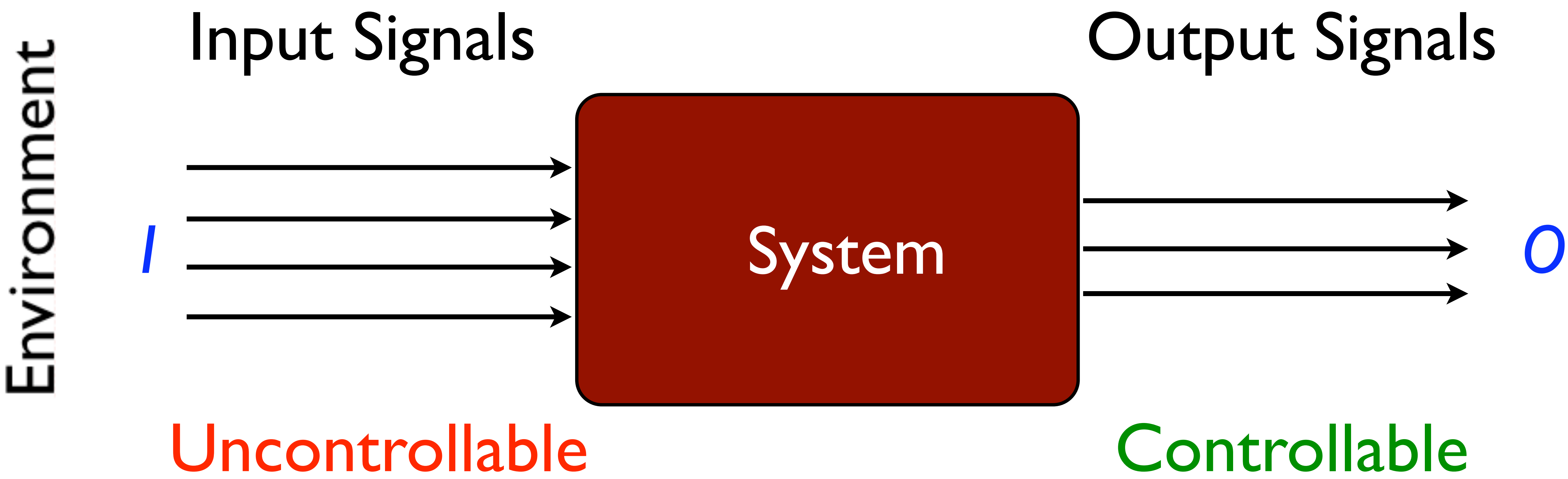
Realizability Problem: Given $\Phi \in$ *LTL on atomic propositions* $I \cup O$

$\exists M \in$ System, $\forall e \in$ Exec, e satisfies Φ ?

LTL Realizability



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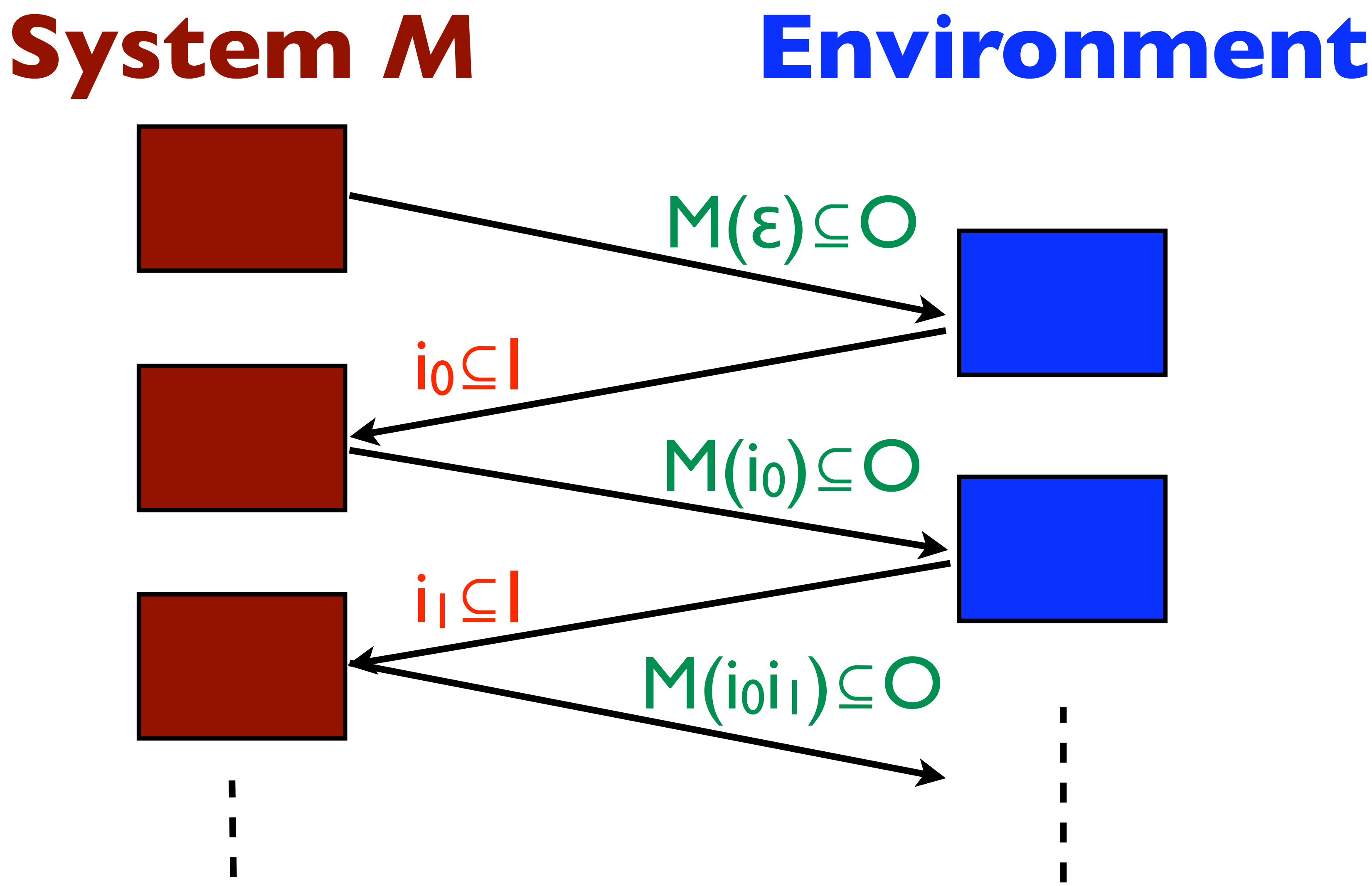
Synthesis Problem: generate such a system

Realizability as an ∞-game



- The system wins the game if the play $(M(\varepsilon) \cup i_0)(M(i_0) \cup i_1)(M(i_0 i_1) \cup i_2)\ldots$ satisfies ϕ
- system ~ *strategy* $(2^I)^* \rightarrow 2^O$

Examples

- $I = \{ i \}$, $O = \{ o \}$

Formula	Satisfiable	Realizable	Strategy
$o \, U \, i$	✔	✘	environment never asserts i
$\Diamond i \rightarrow o \, U \, i$	✔	✔	system always asserts o

Existing Procedures

- 2ExpTime-Complete [Rosner, 92]
- "classical" procedure [Pnueli, Rosner, 89]

LTL $\xrightarrow{2^{O(n)}}$ Büchi Word Automata $\xrightarrow{2^{O(m \log m)}}$ **Det**. Rabin Word Automata

[Safra, 88]

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Solve a Rabin Game

in Word Automata

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LTL $\xrightarrow{2^{O(n)}}$ Büchi W ... in Word Automata

Solve a Rabin Game

- Safraless procedure [Kupferman, Vardi, 05]

LTL → Universal CoBüchi Tree Automata

↓

Büchi Tree Automata ← Alternating Weak Tree Automata

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Büchi Tree Autom … ee Automata

Implemented in *Lily* [Jobstmann, Bloem,06]

A New Safraless Approach

LTL

↓ $2^{O(n)}$

Universal coBüchi Word automata

↓ $O(1)$

Universal **KcoBüchi** Word automata

↓ $2^{O(m^2)}$

Det. KcoBüchi Word automata

Universal **KcoBüchi** Word: all run visit at most **K** accepting states

A New Safraless Approach

LTL

$2^{O(n)}$ ↓

Un … ata

Solve a Safety Game

Univ … mata

$2^{O(m^2)}$ ↓

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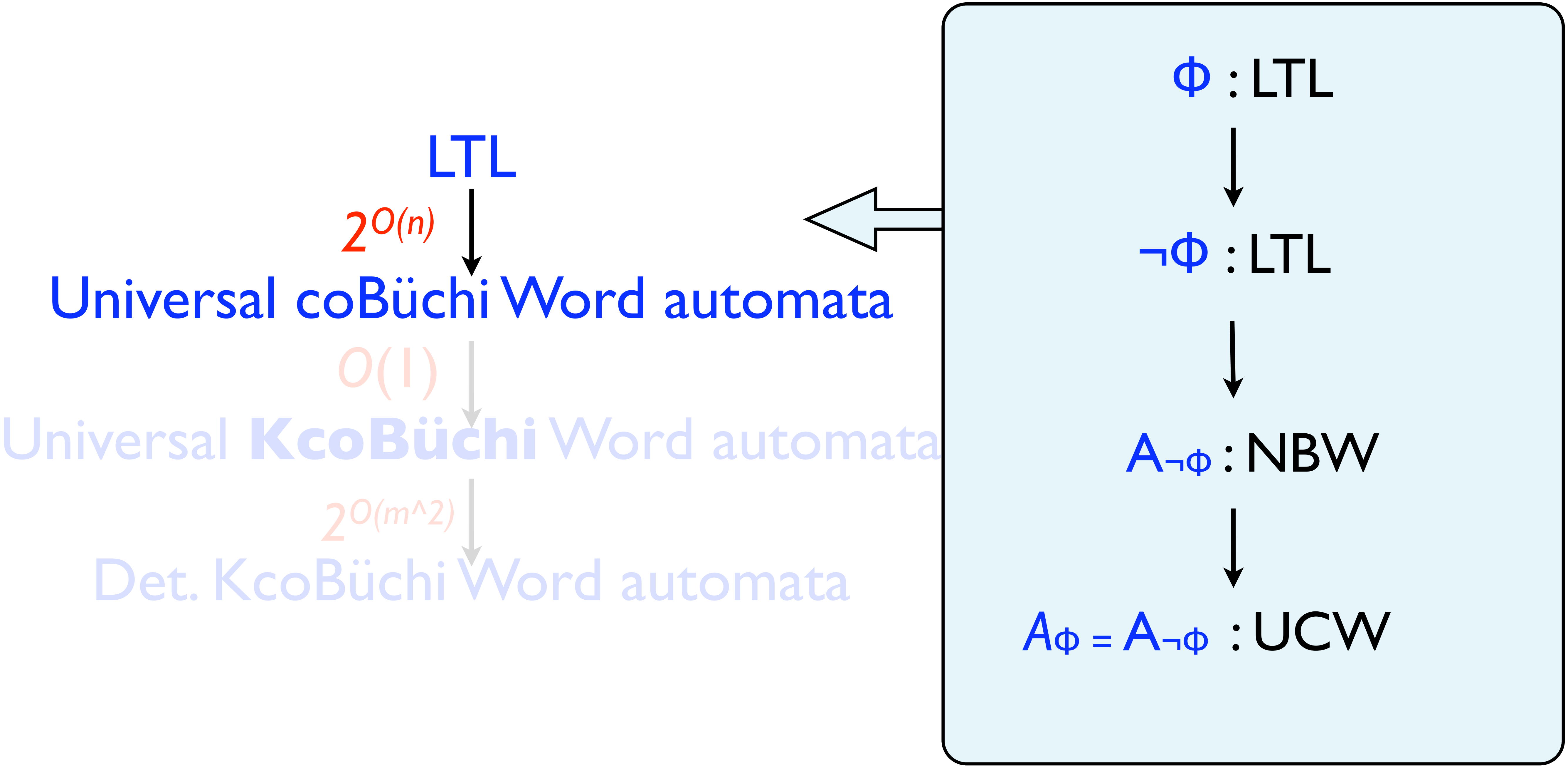
Doubly Exponential Size ...

... Game solved on-the-fly with **antichain** technics

Universal **KcoBüchi** Word: all run visit at most **K** accepting states

A New Safraless Approach



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Det. KcoBüchi Word automata

Theorem [Safra88,Kupferman-Vardi05]

Let A: UCW with n states,

A is realizable

$\longleftrightarrow$

it is realizable by a finite-state strategy S with at most n^{2n+1} states.

Consequence

the runs of A on words compatible with S visits at most **K**$=n^{2n+2}$ final states

Universal **KcoBüchi** Word: all run visit at most **K** accepting states

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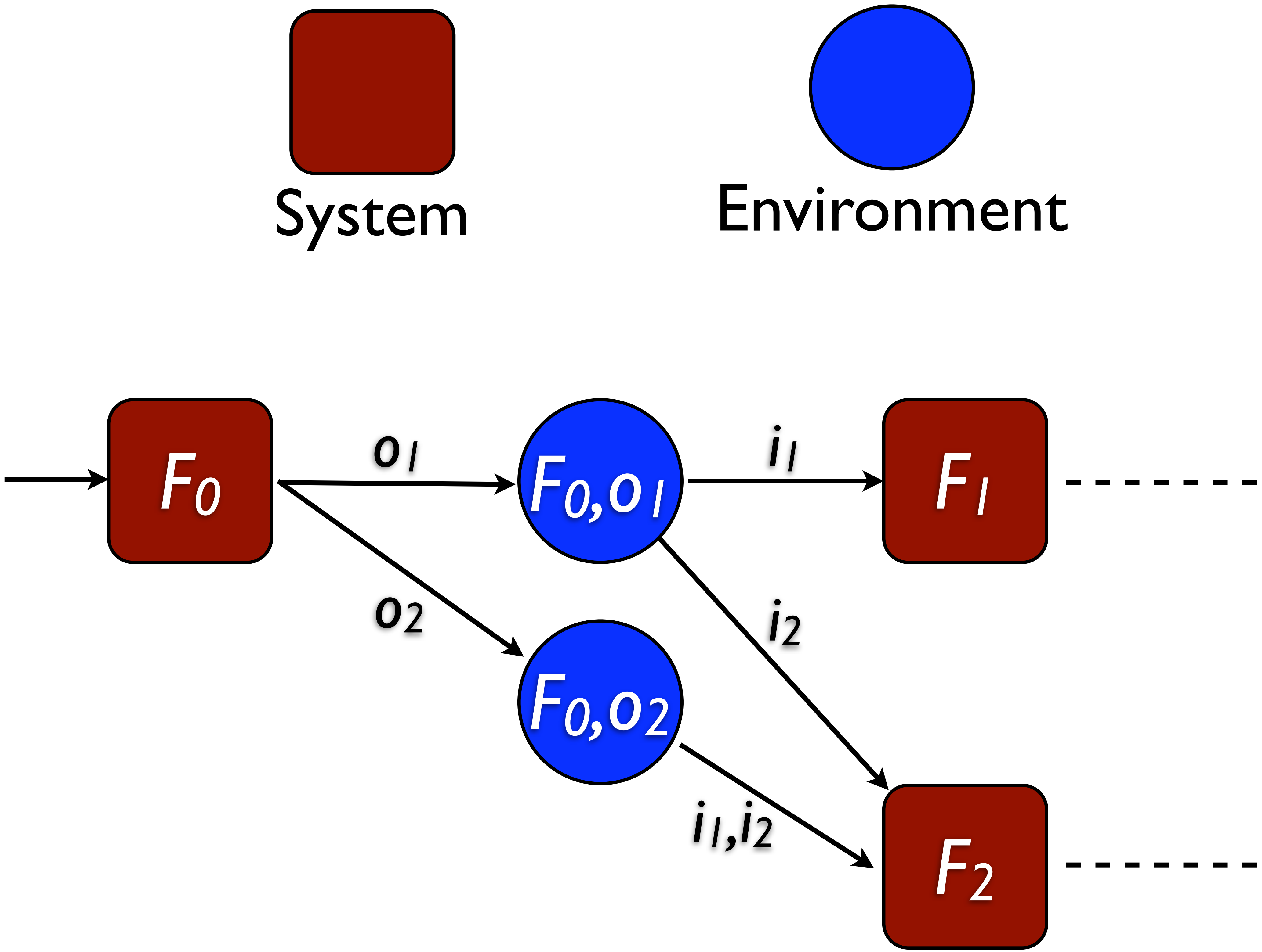
Det. KcoBüchi Word automata

Determinization

- For each state *q*, count the maximal number of final states visited by runs ending up in *q*
- Set of states: *counting functions F* from *Q* to [-1,0,...,**K**+1]
- Final states are functions *F* such that ∃*q*: *F*(*q*)>**K**
- set the bound to *0*

Universal **KcoBüchi** Word: all run visit at most **K** accepting states

Realizability as a Safety Game



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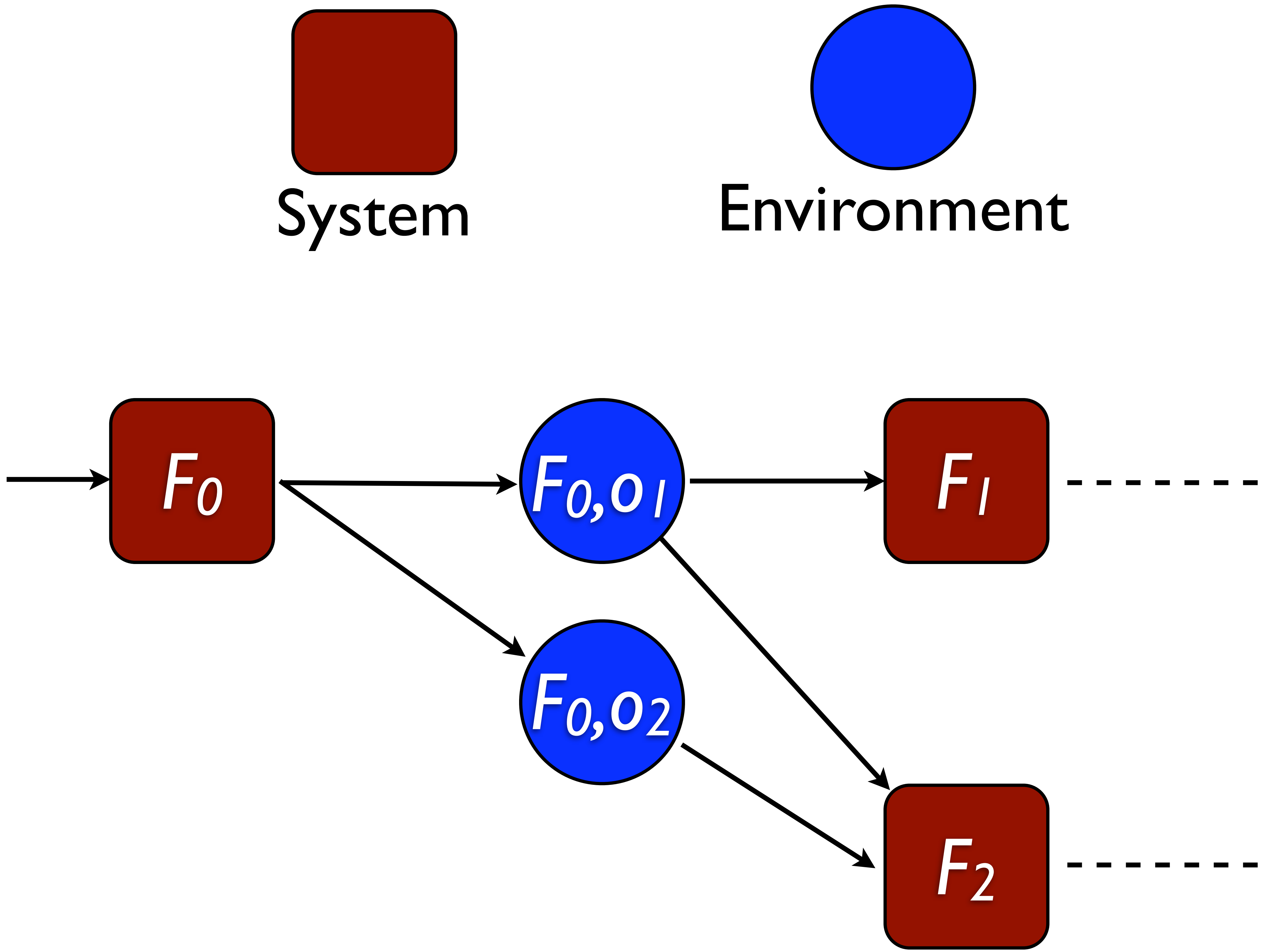

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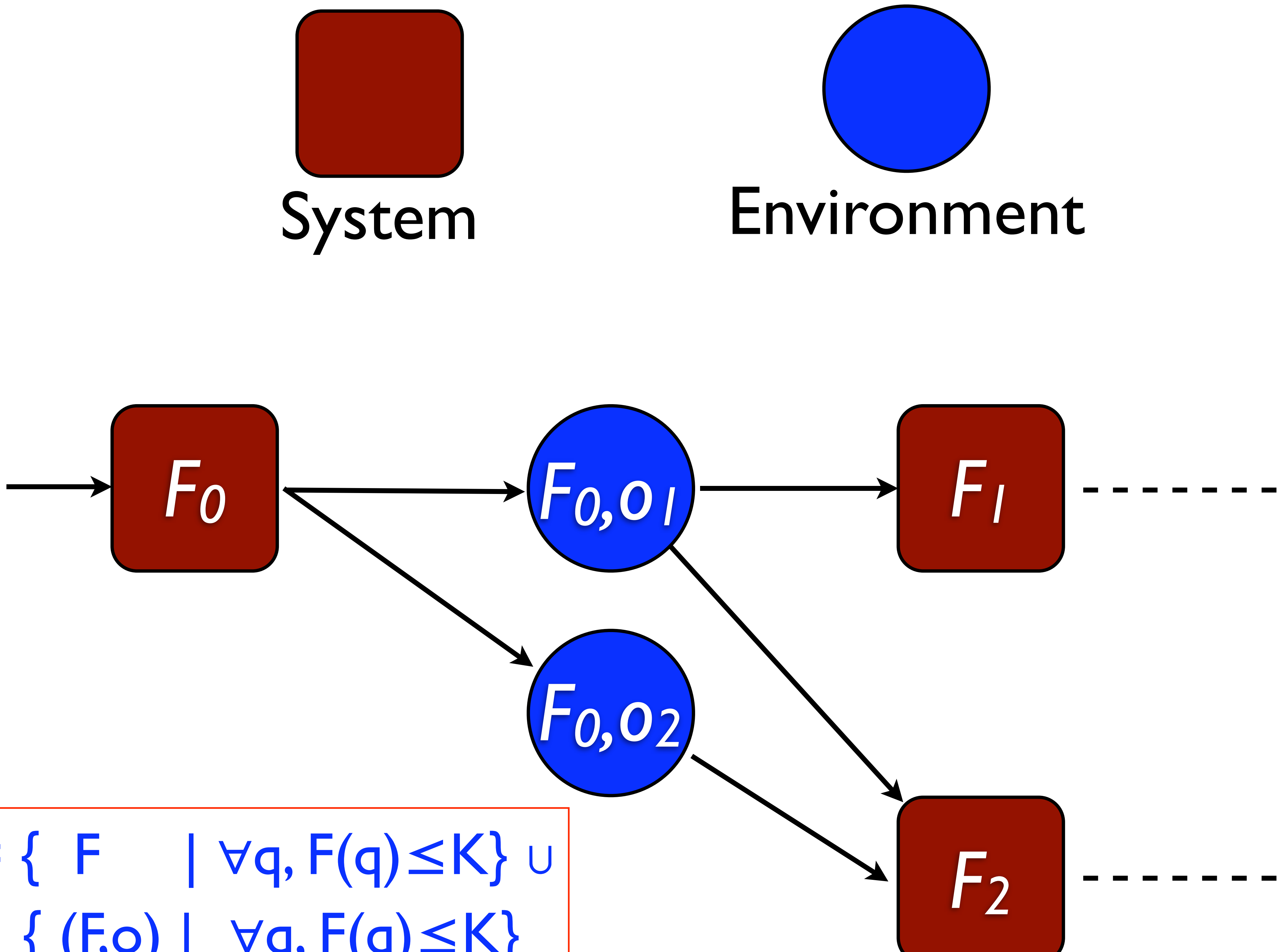
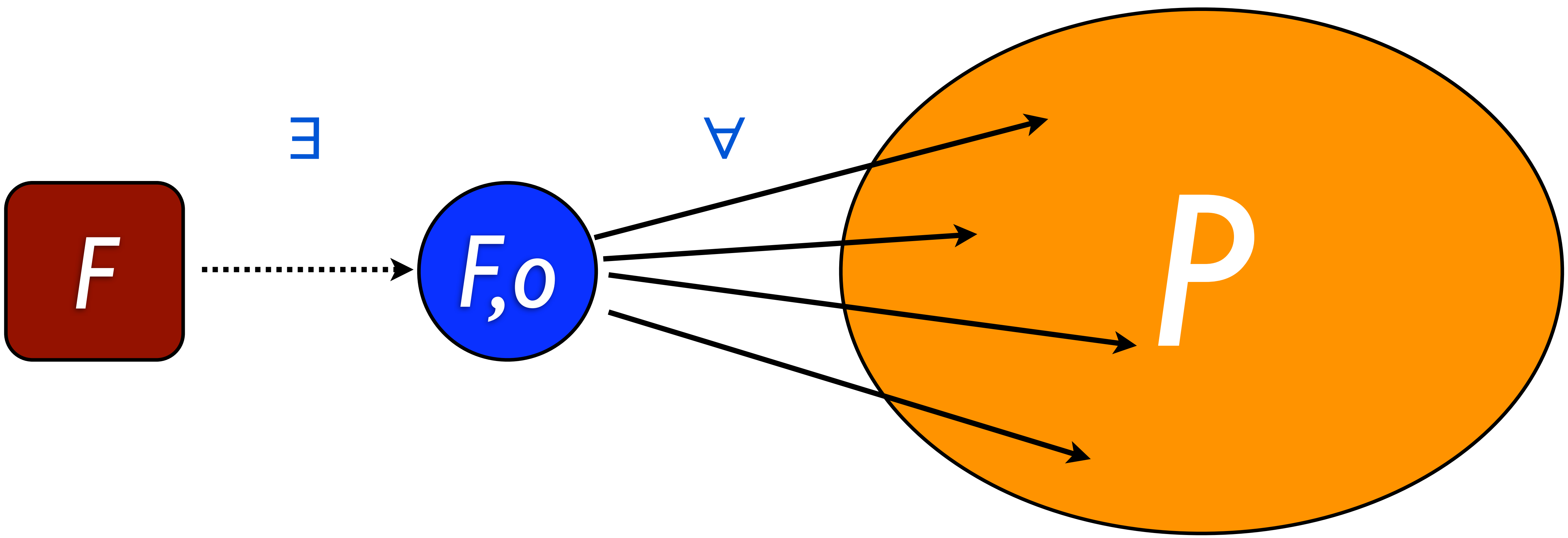


$$\text{Safe} = \{\ F \mid \forall q, F(q) \leq K\} \cup \{\ (F,o) \mid \forall q, F(q) \leq K\}$$

Controllable Predecessors

- $P \subseteq \mathbb{F}$: subset of system positions
- safe controllable predecessors of P
  $Pre(P) = \{ F \mid \exists o \subseteq O, \forall F', ((F,o),F') \in T \Rightarrow F' \in P\} \cap Safe$



- greatest fixpoint Pre^* = *winning region for System*

Controllable Predecessors

1. partial order on counting functions:

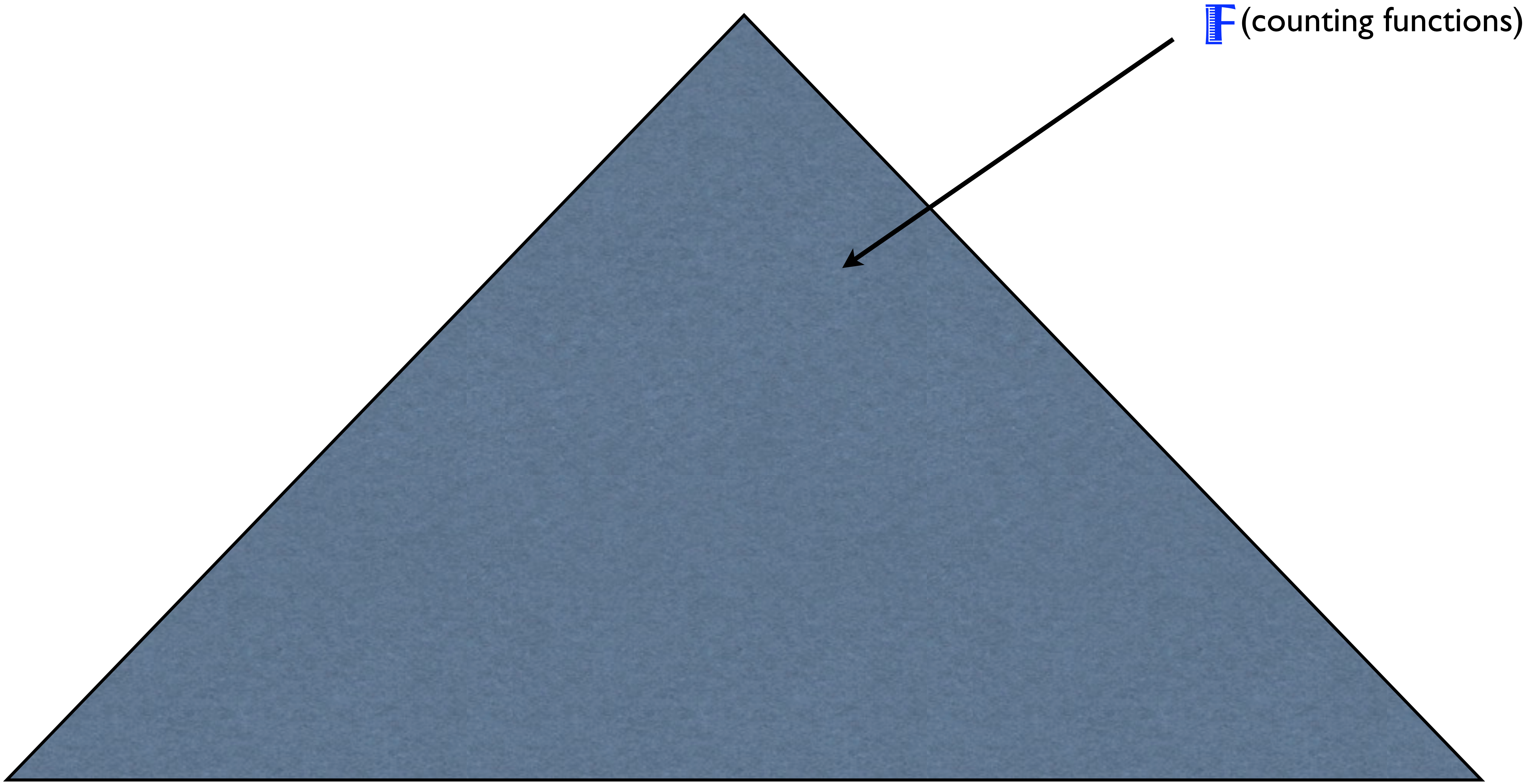
$$F \leq_d F' \text{ if } \forall q\text{: } F(q) \leq F'(q)$$

2. if System wins from *F'*, she also wins from

3. *Pre(.)* preserves *downward*-closed sets

4. represent each (downward) set of the fixpoint computation by its maximal elements

Symbolic Fixpoint Computation



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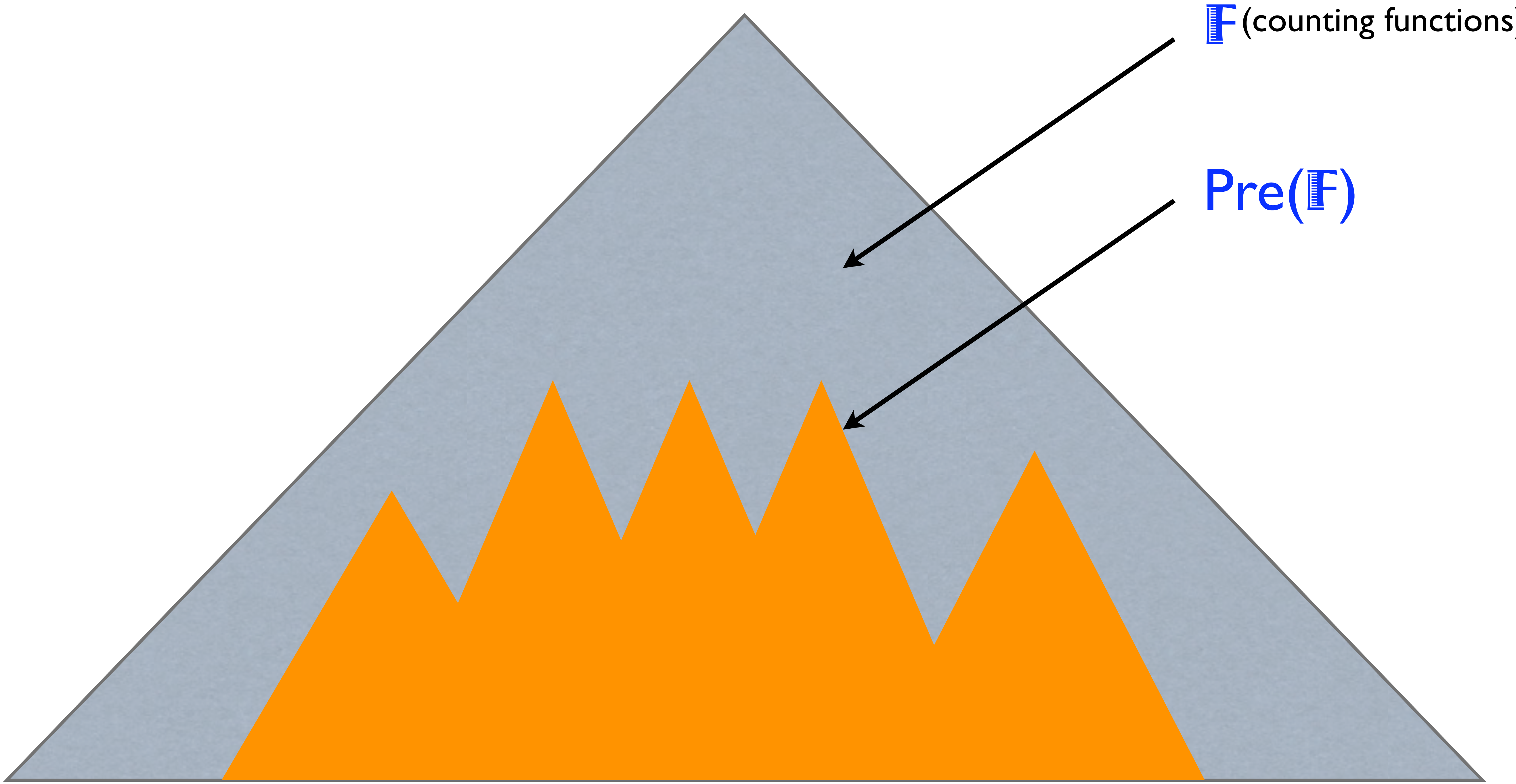

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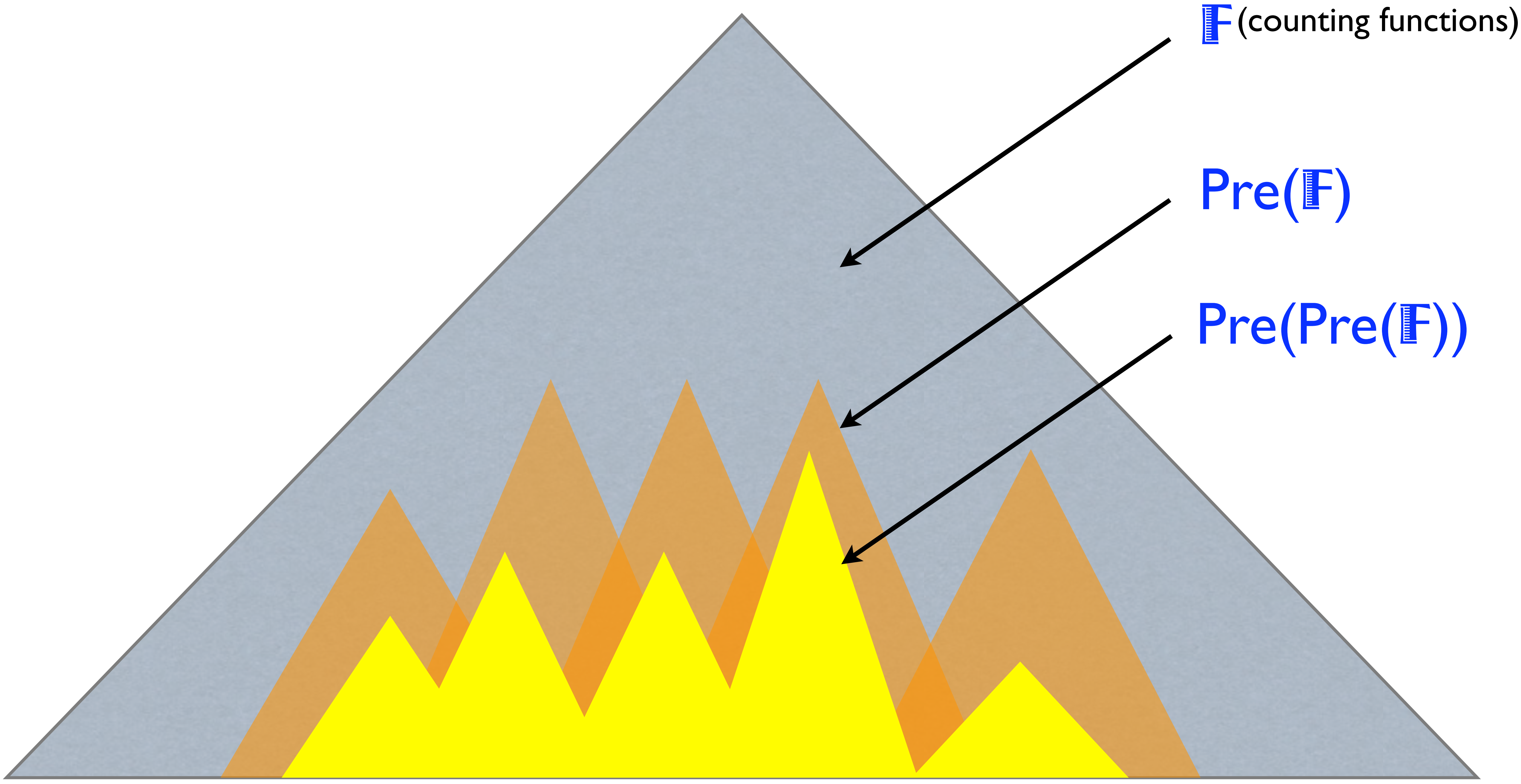

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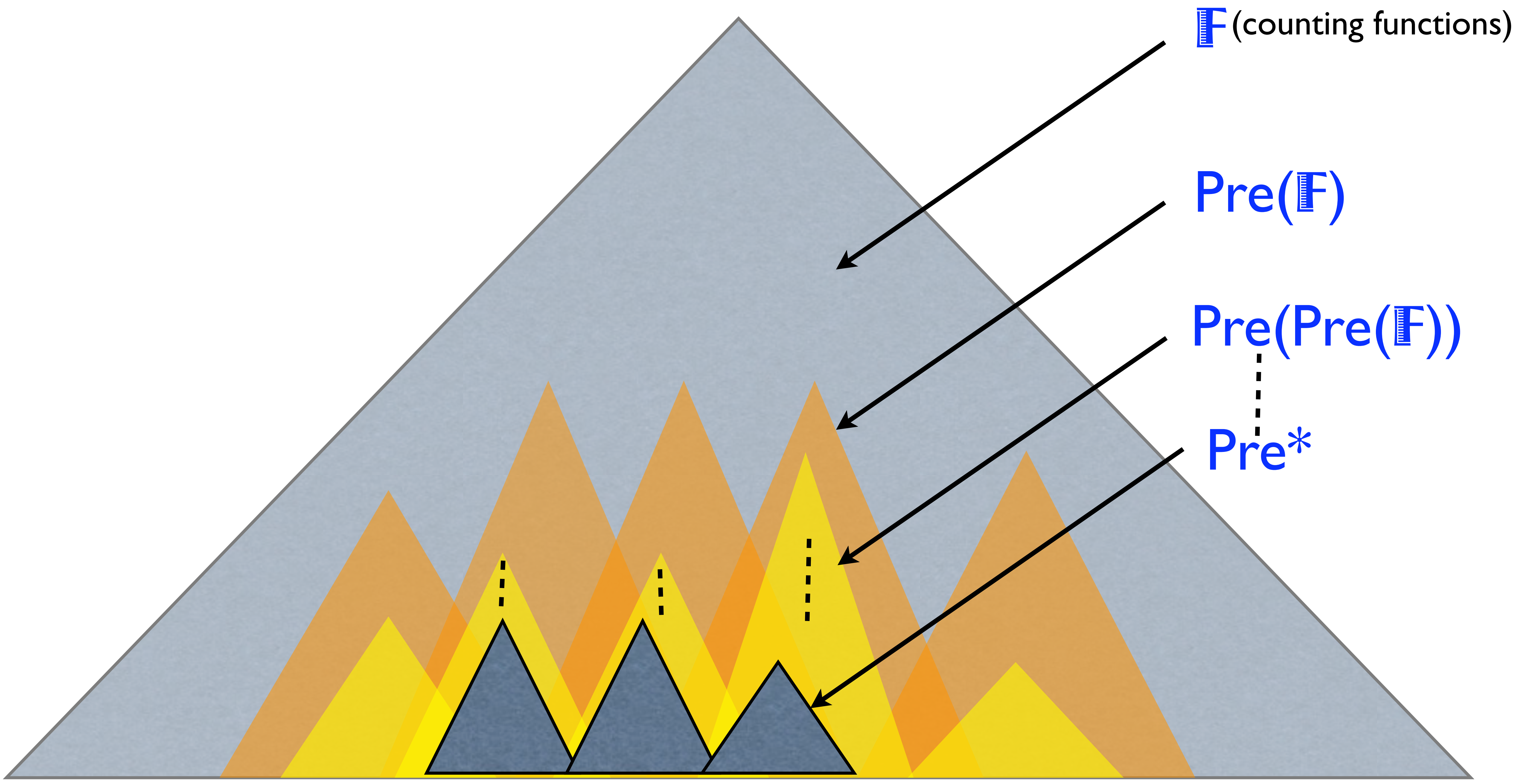

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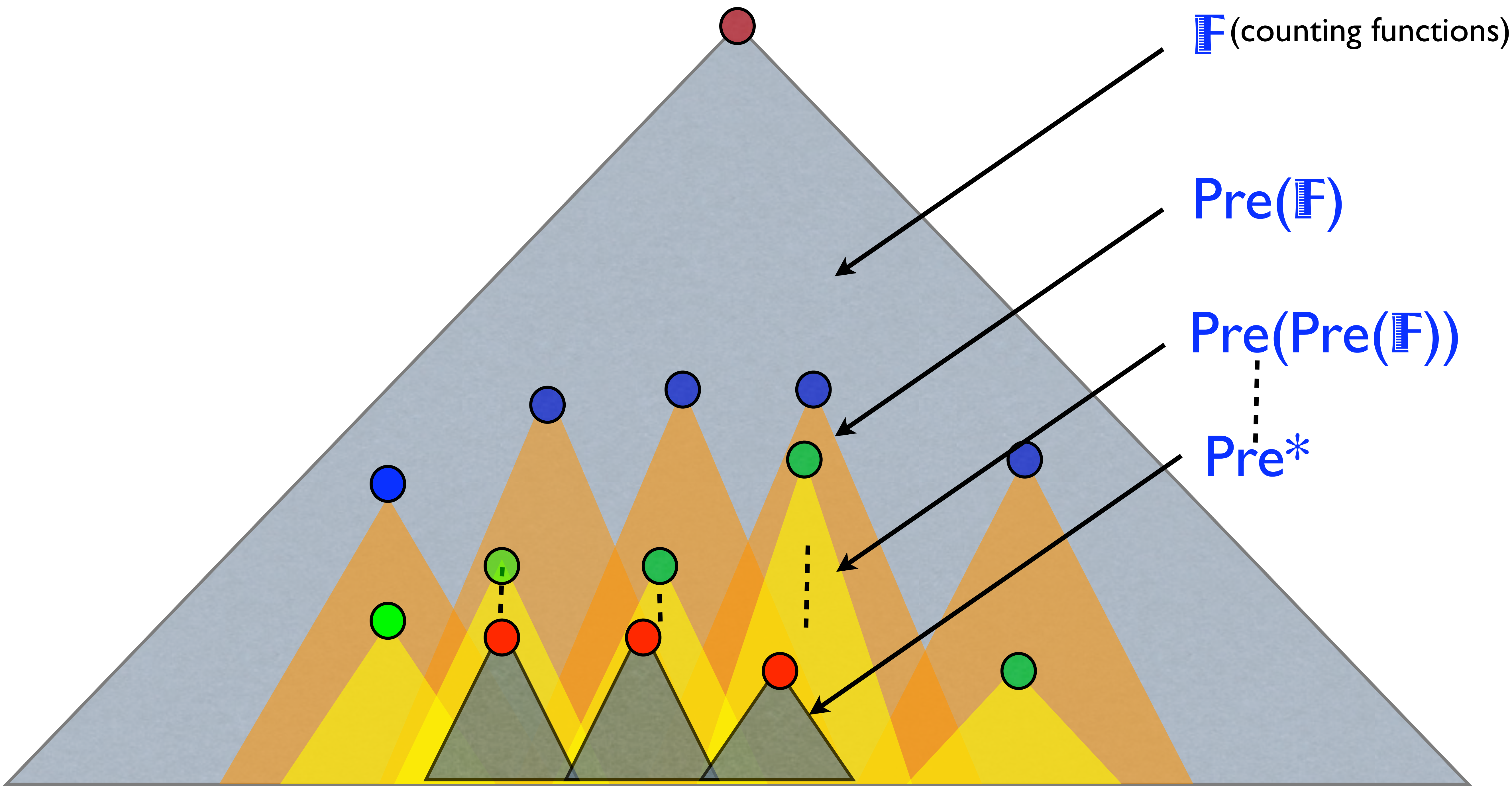

Incremental Algorithm

- the bound **K** is very big (doubly exponential)
- if the spec is realizable with a "small" bound, it is realizable with a "big" bound
- iterate over *k=0,1,...,***K**

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Not reasonable for unrealizable specifications

Incremental Algorithm

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Not

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But by Martin’s determination theorem:

ϕ is unrealizable for the System iff $\neg\phi$ is realizable for the Environment.

Experiments

- implementation in Perl (as Lily)
- if the spec is realizable, output a Moore machine that realizes it
- formula to automata construction borrowed from Lily (based on Wring [Somenzi, Bloem])
- significantly faster on all realizable Lily's examples
- bottleneck: formula to automaton construction

Future Work ...

- compositionnality
- avoid automata construction to handle larger formulas

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... Thank You