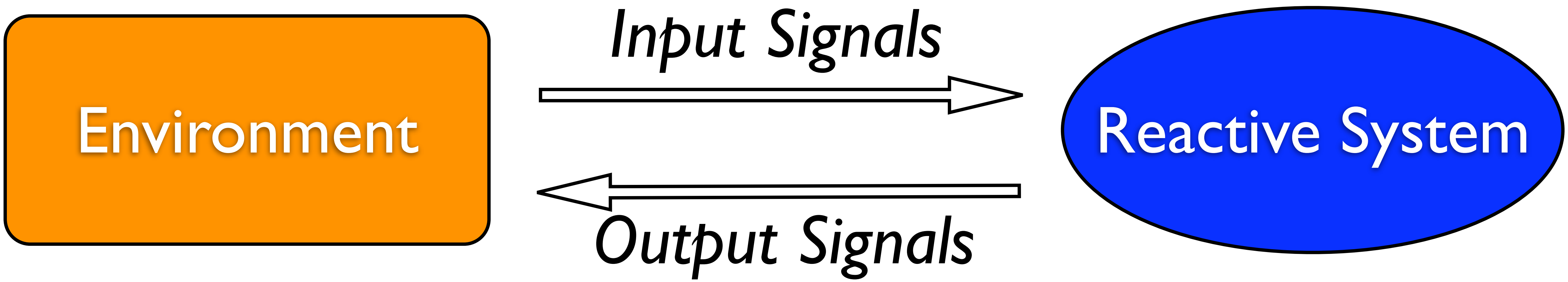
Towards Efficient Synthesis of LTL Specifications

Emmanuel Filiot
joint with Naiyong Jin and Jean-François Raskin

Université Libre de Bruxelles

FNRS contact day

Reactive Systems



- continuous interaction with their environment
- non-terminating
- have to respect real-time properties (e.g. safety properties)
- have to cope with the uncontrollable behavior of their environment

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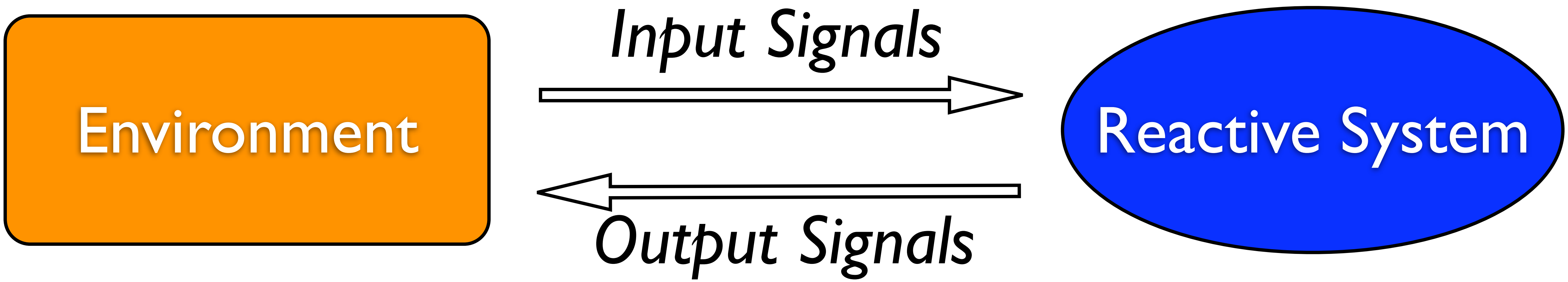

- continuous interaction with their environment
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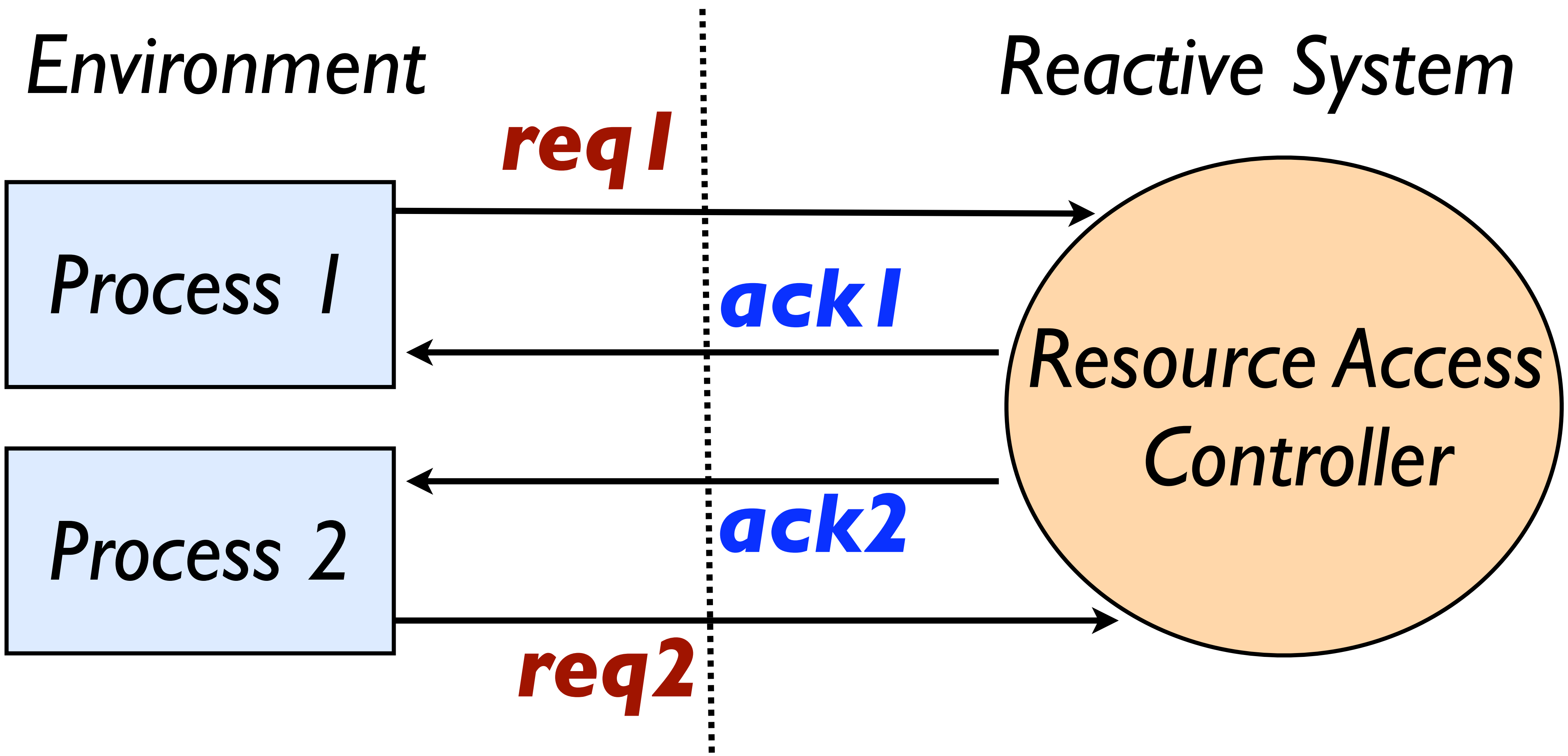
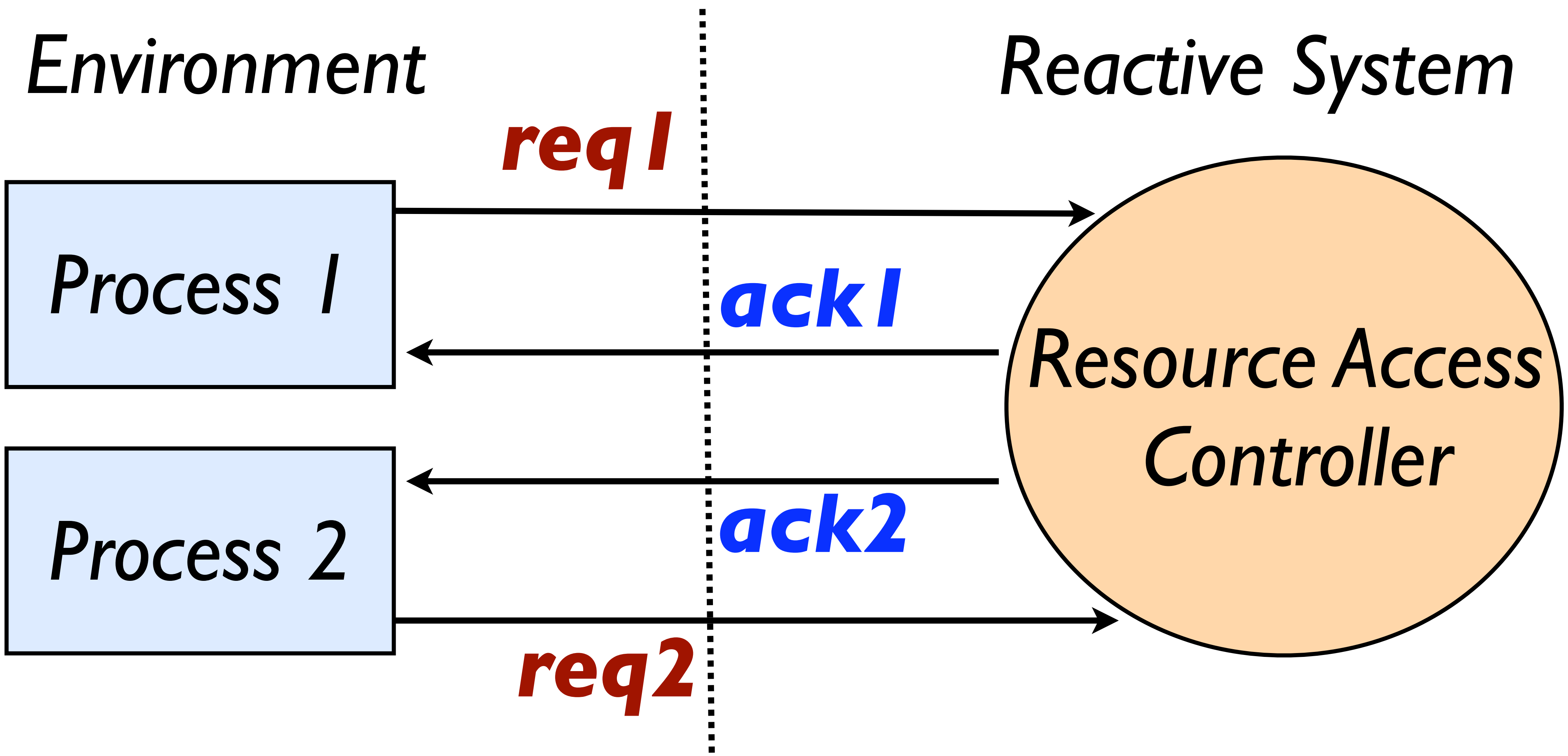
- have to cope with the uncontrollable behavior of their environment

Hard to design, needs **synthesis** from specification!

Example


Environment
Reactive System
req1
Process 1
ack1
Resource Access Controller
Process 2
ack2
req2

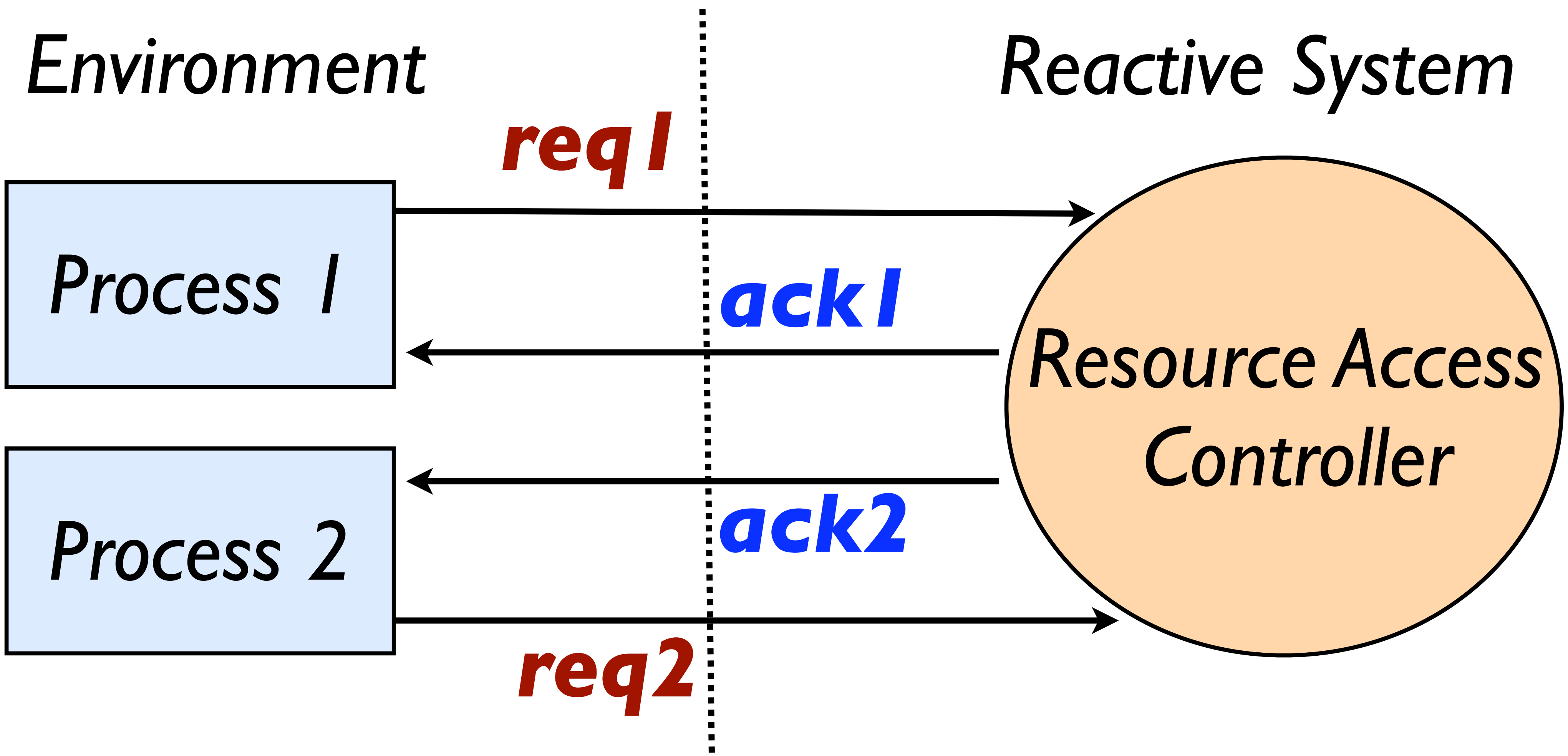
Example



Executions: infinite sequences of sets of signals

{***req1***, ***req2***} {***ack1***} {***req1***} {***ack2***} {***req1***} {***ack1***} **...**

Example



Properties we would like to ensure

Liveness property: G (**reqi** -> F **acki**) ***i=1,2***

Safety property: G (¬**ack1** ∨ ¬**ack2**)

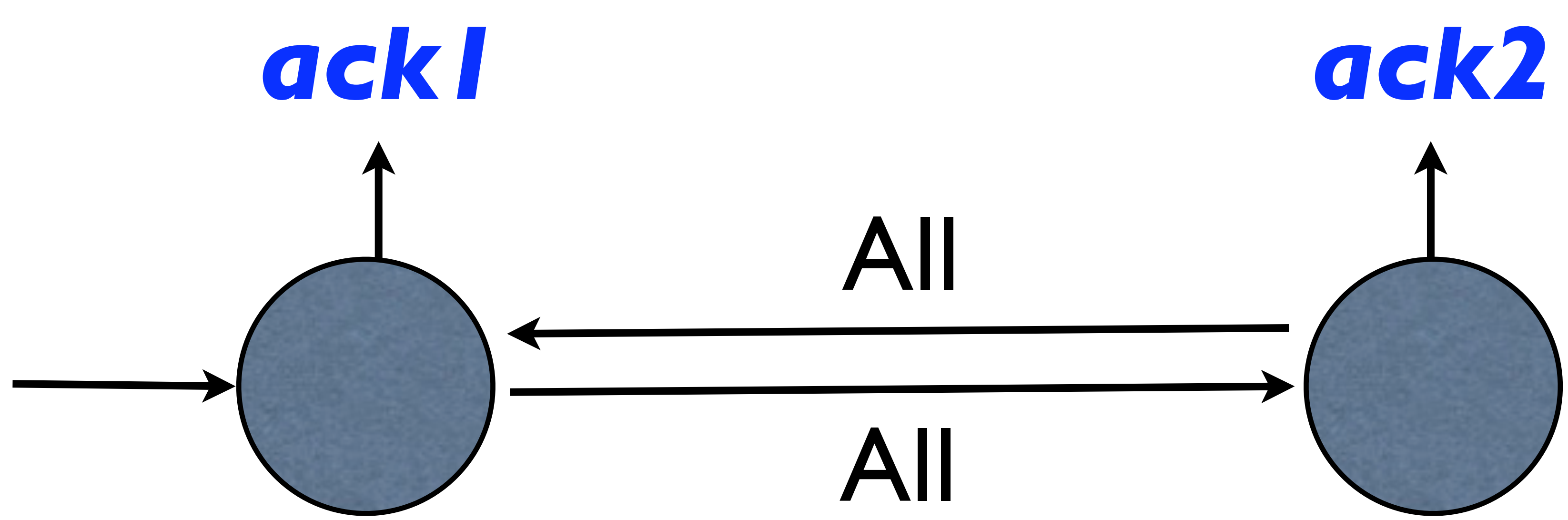
LTL Synthesis

LTL Spec

Liveness property: G (***reqi*** -> F ***acki***) ***i=1,2***

Safety property: G (¬***ack1*** ∨ ¬***ack2***)

generate a RS that realizes the spec



LTL Synthesis

LTL Spec

Liveness property: G (***reqi*** -> F ***acki***) ***i=1,2***

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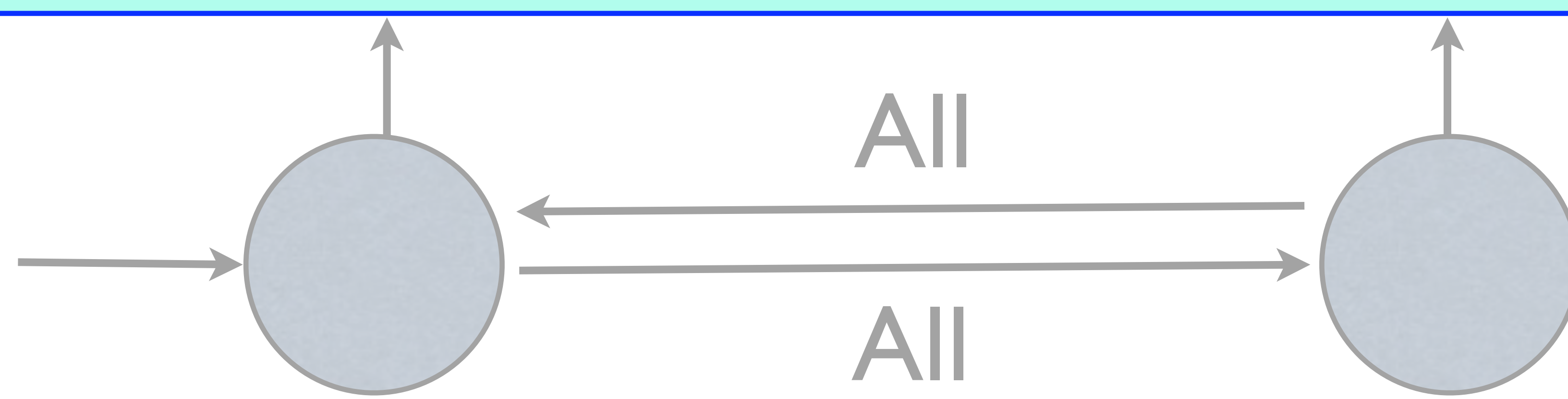
Realizability

Given an LTL spec, does there exists a RS such that all its executions (whatever the environment does) satisfy the spec ?

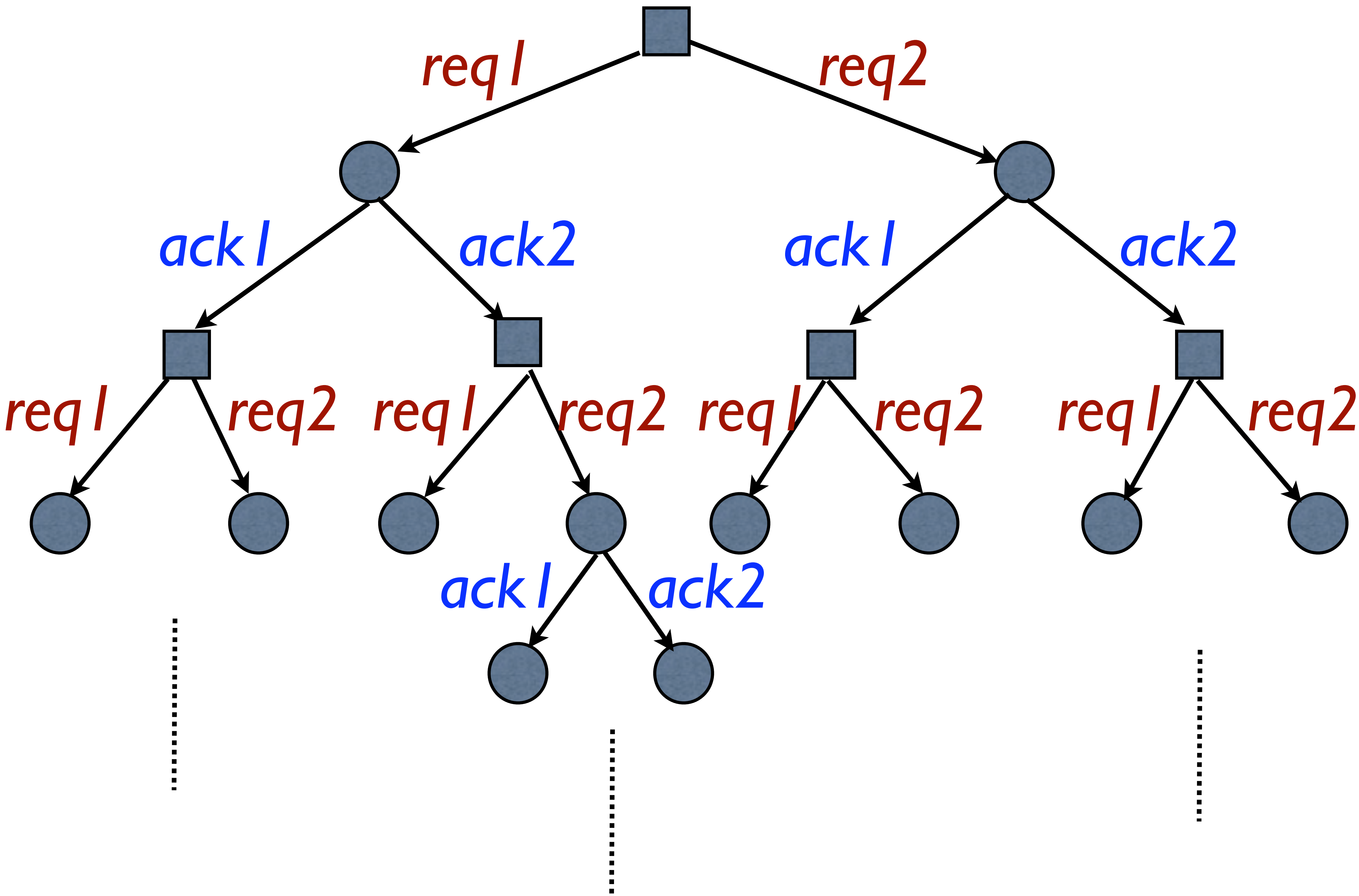


Unrealizable Spec

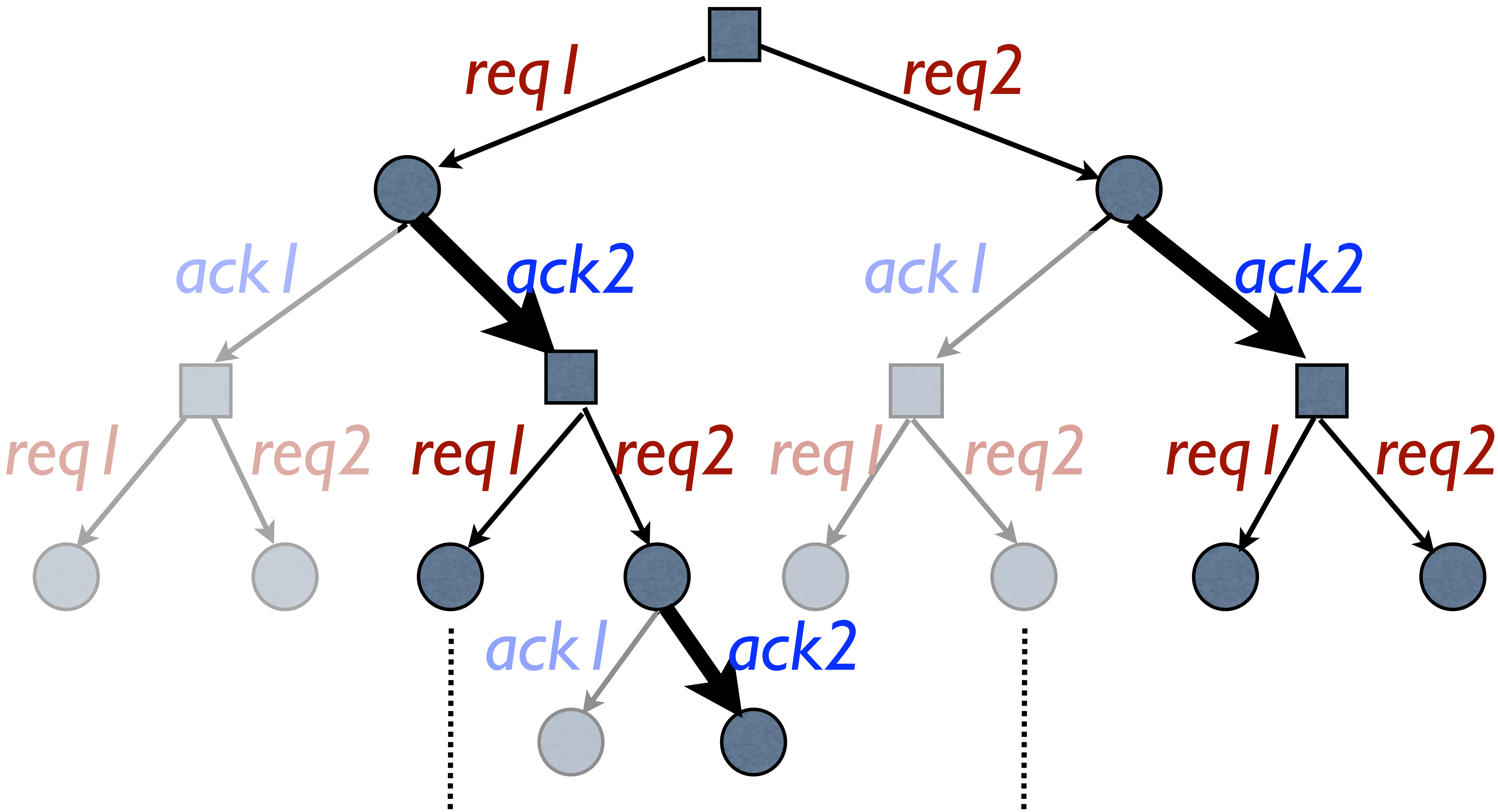
G (ack1 -> F req1)

"Each time the system acknowledge, the environment eventually sends a request"

Synthesis as Game



Reactive System as a Strategy



All the infinite paths have to satisfy the spec

Existing Procedures

- if a spec is realizable, it is realizable by a finite-steate strategy
- 2ExpTime-Complete [Rosner, 92]
- "classical" procedure [Pnueli, Rosner, 89]

LTL $\longrightarrow$ **Rabin Game**

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- Safraless procedure [Kupferman, Vardi, 05]

LTL $\longrightarrow$ **Büchi Game**

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Implemented in Lily [Jobstmann, Bloem]

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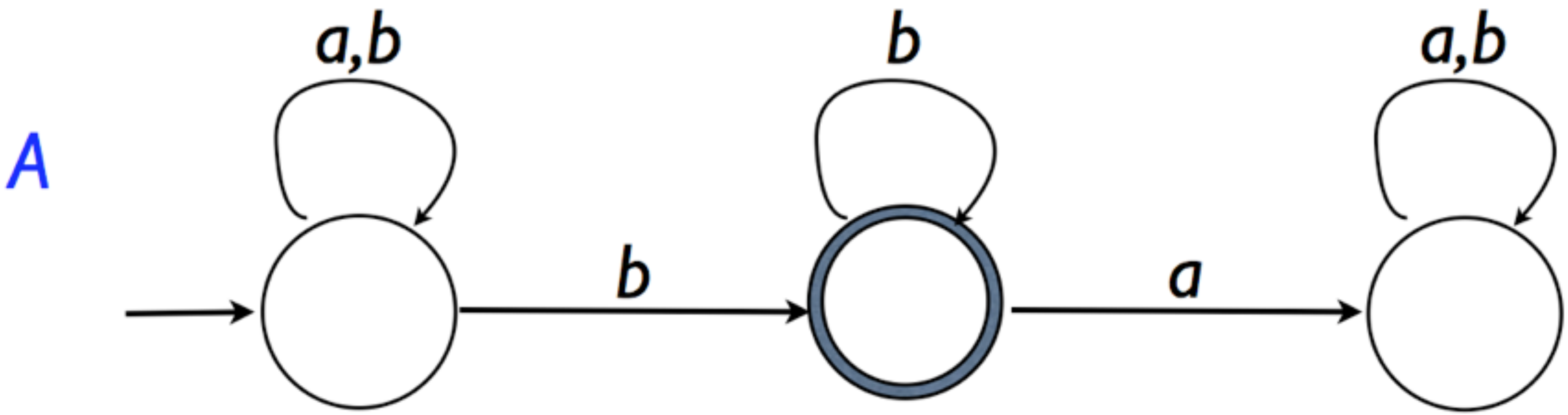
Implemented in Lily [Jobstmann, Bloem]

- in this talk: LTL $\longrightarrow$ Safety Game

K-CoBüchi Automata



An (infinite) word is accepted

iff

all its runs visits **at most *K*** accepting states

In this example: words with at most K symbols b

Automata as a spec

- K-co-Büchi automata specify infinite words
- they can be used as RS specifications

Theorem

For any LTL specification Φ, one can construct a K-co-Büchi automaton A such that:

Φ is realizable

iff

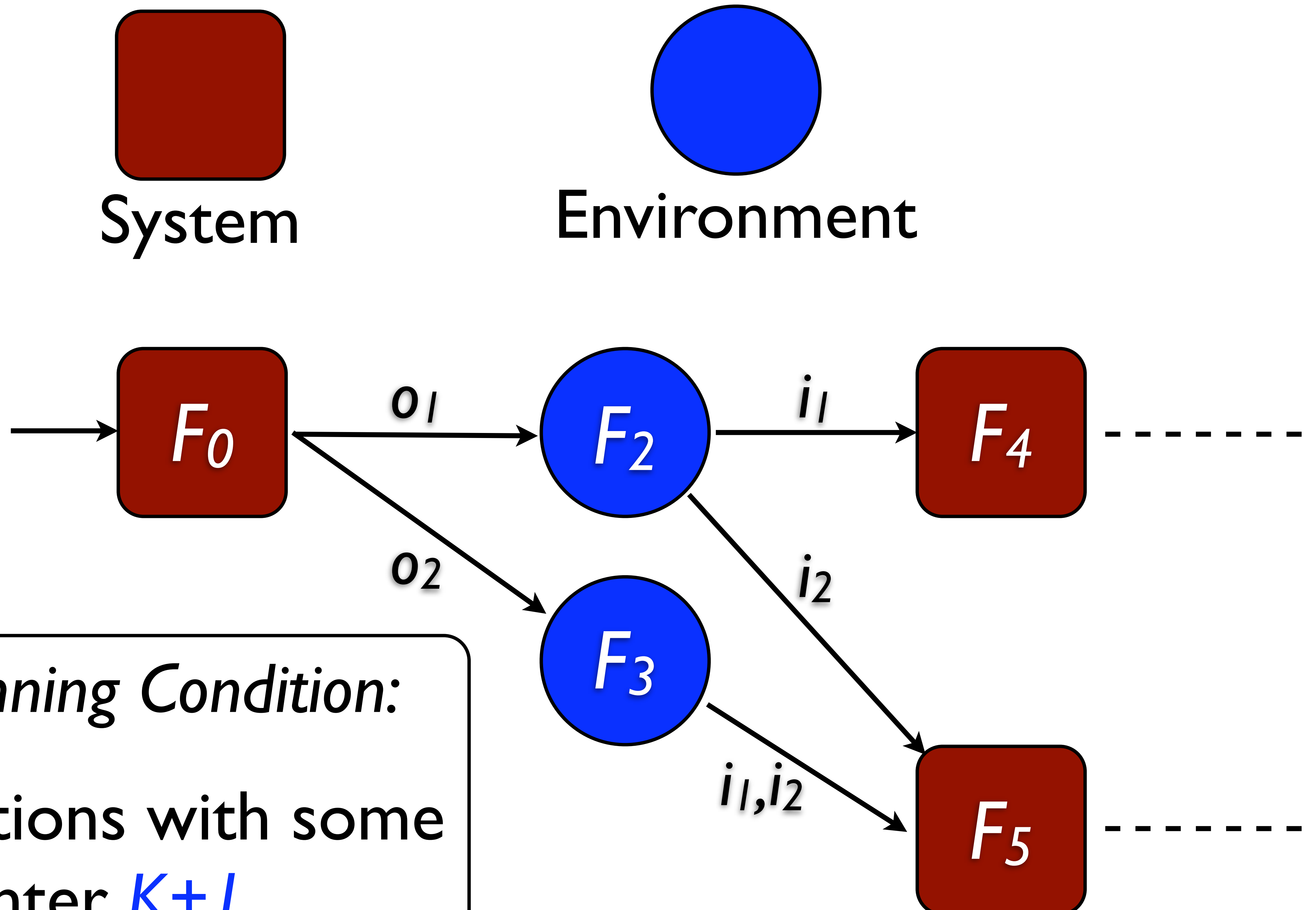
A is realizable

Construction: exptime and $K = O(2^{|\Phi|})$

Determinization

- *K*-Co-Büchi automata are easily determinizable
- extend subset construction with counters (up to $K+1$)
- states: Functions $F: Q \longrightarrow \{0,1,\ldots,K+1\}$

Playing on automata



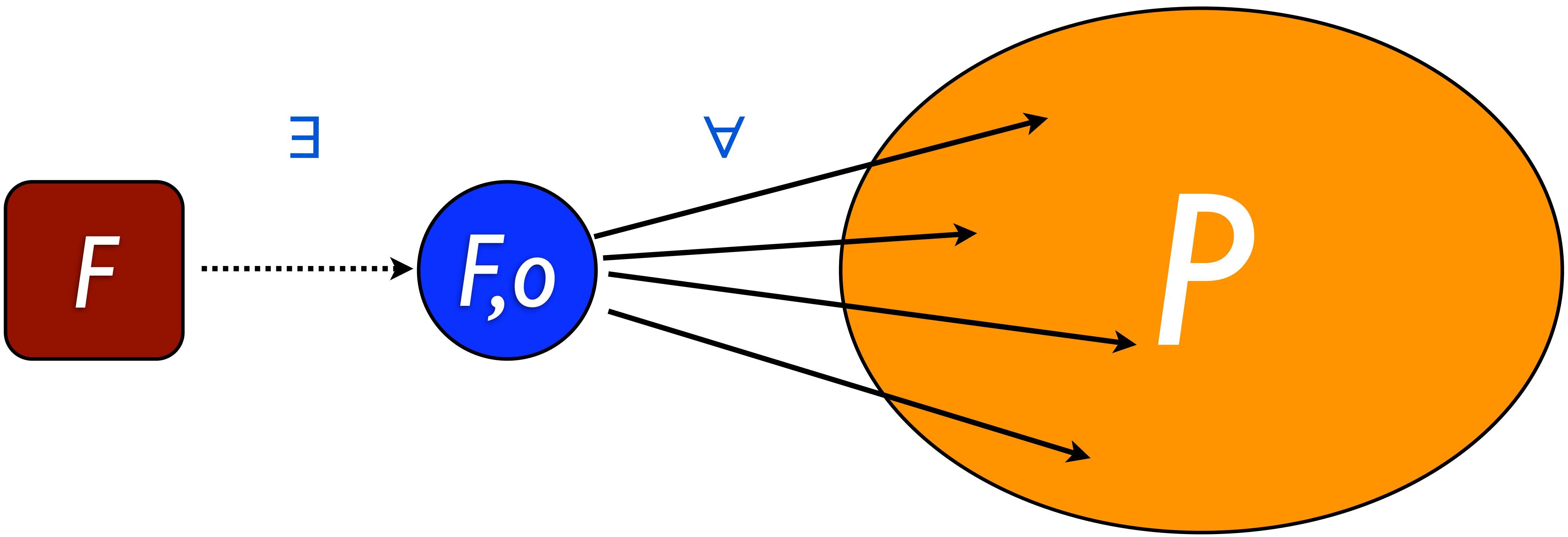
Safety Winning Condition:

Avoid functions with some counter $K+1$

Controllable Predecessors

- $P \subseteq \mathbb{F}$: subset of system positions

- safe controllable predecessors of P
  $Pre(P) = \{ F \mid \exists o \subseteq O, \forall F', ((F,o),F') \in T \Rightarrow F' \in P \}$



- greatest fixpoint Pre^* = *winning region for System*

Controllable Predecessors

1. partial order on counting functions:

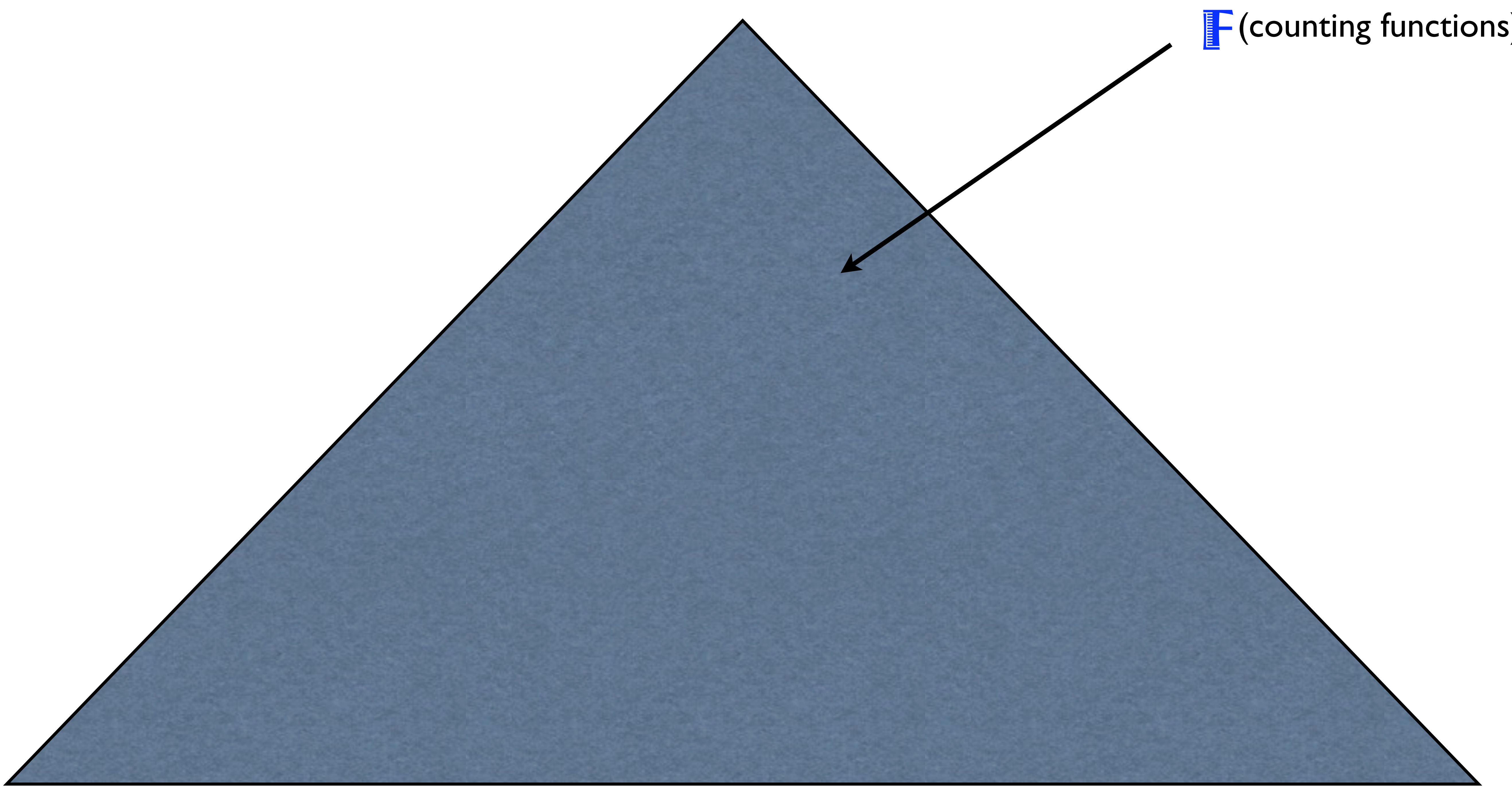
$$F \leq_d F' \text{ if } \forall q\text{: } F(q) \leq F'(q)$$

2. if System wins from *F'*, she also wins from

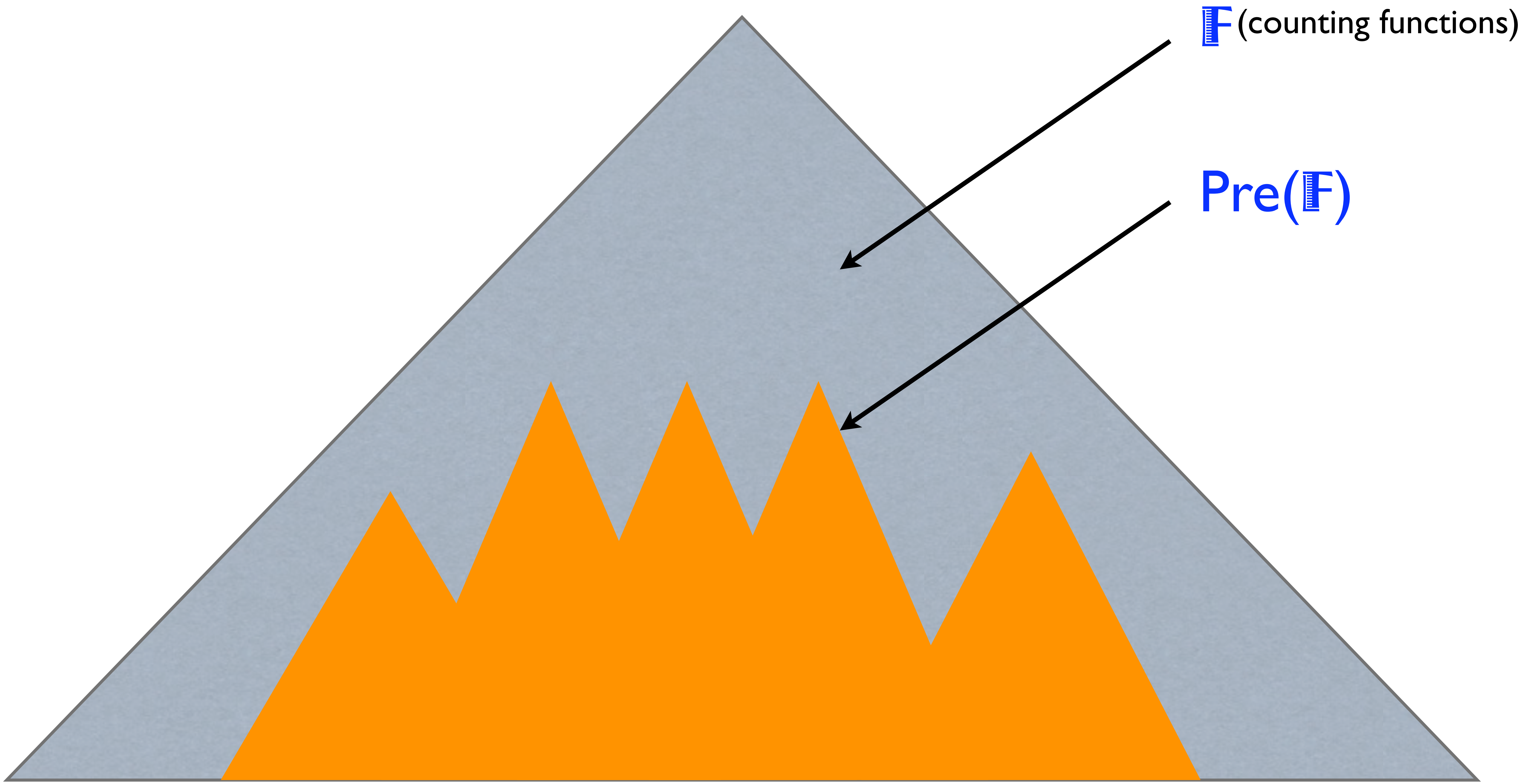
3. *Pre(.)* preserves *downward*-closed sets

4. represent each (downward) set of the fixpoint computation by its maximal elements

Symbolic Fixpoint Computation



Symbolic Fixpoint Computation



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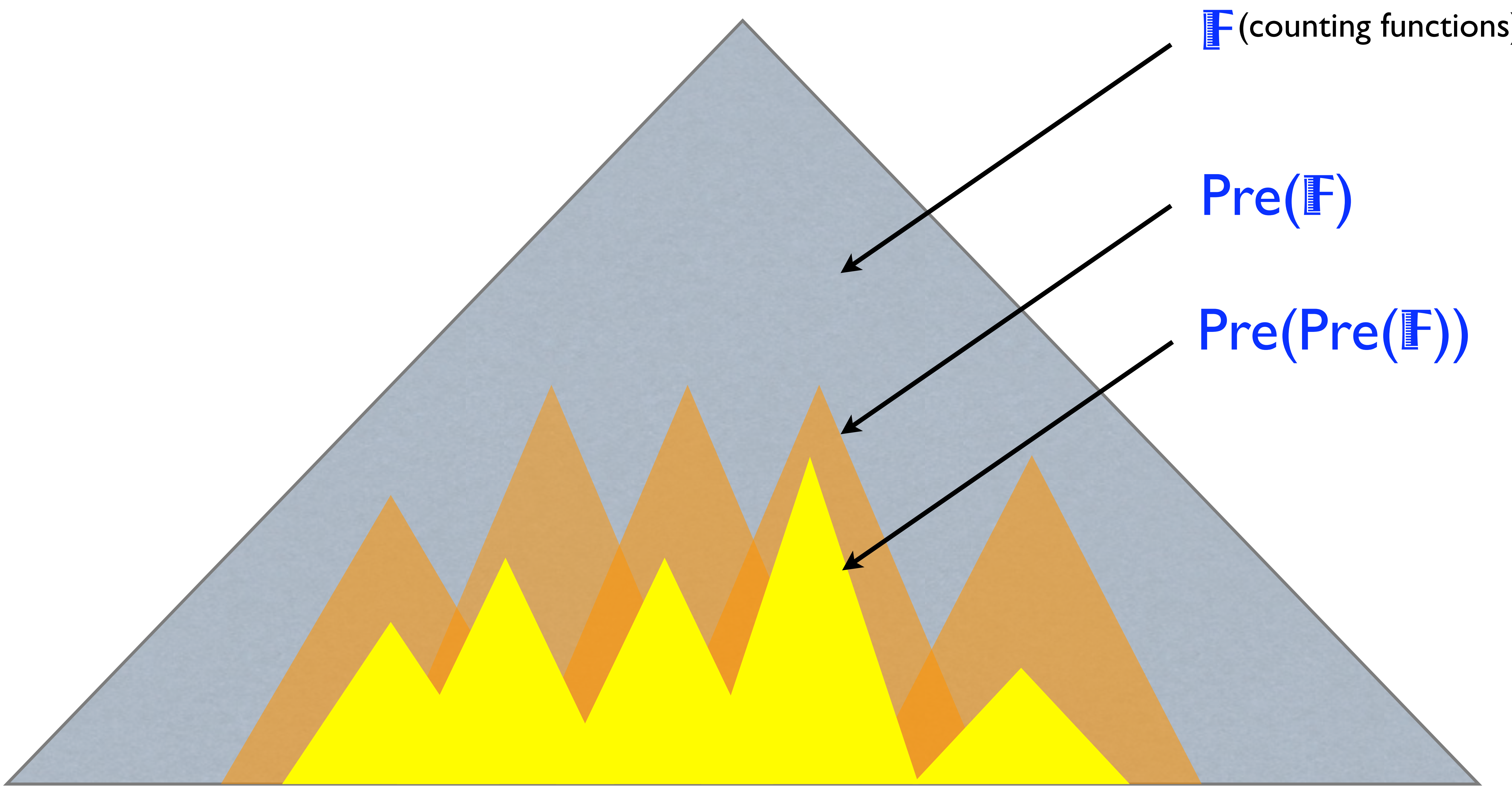

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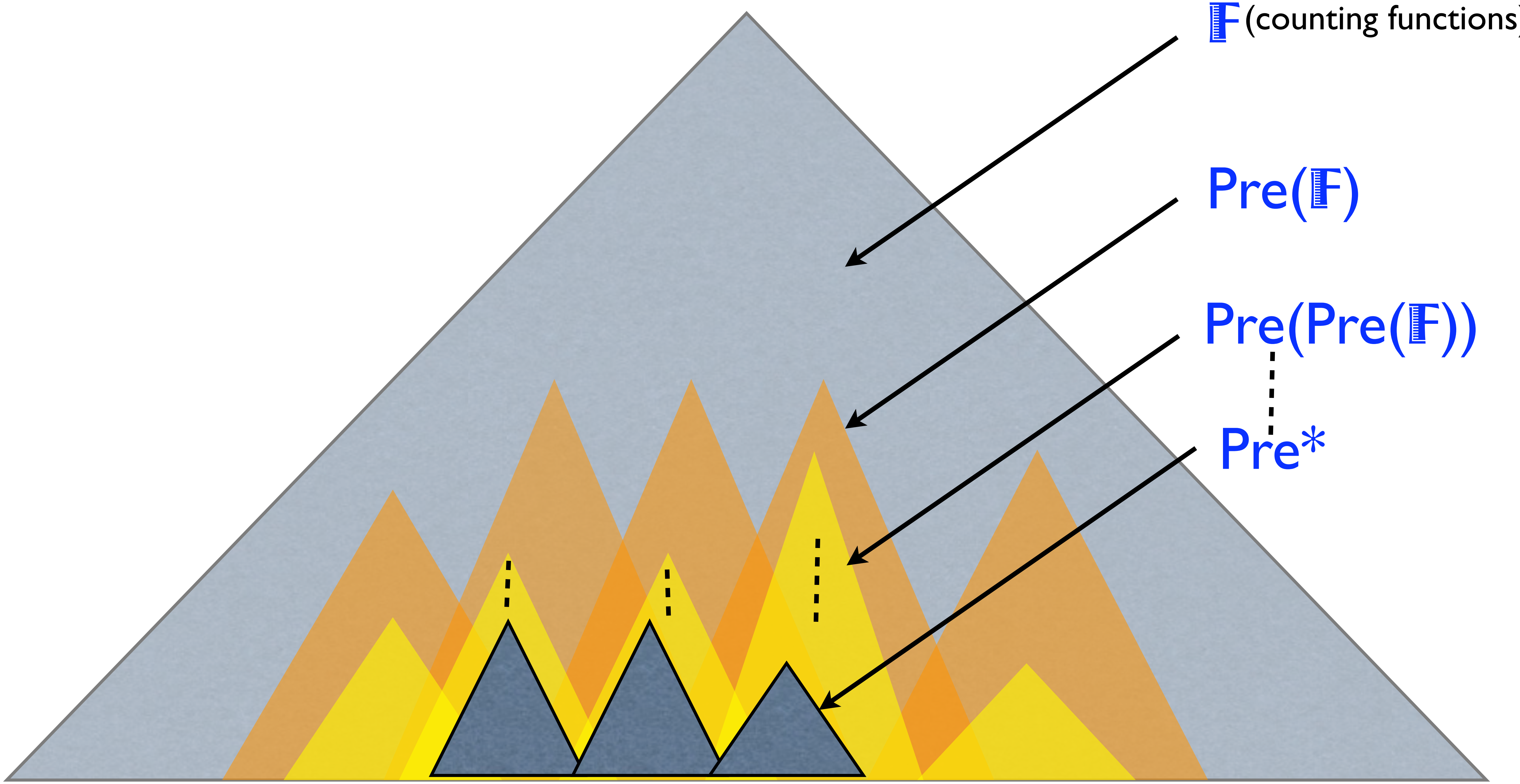

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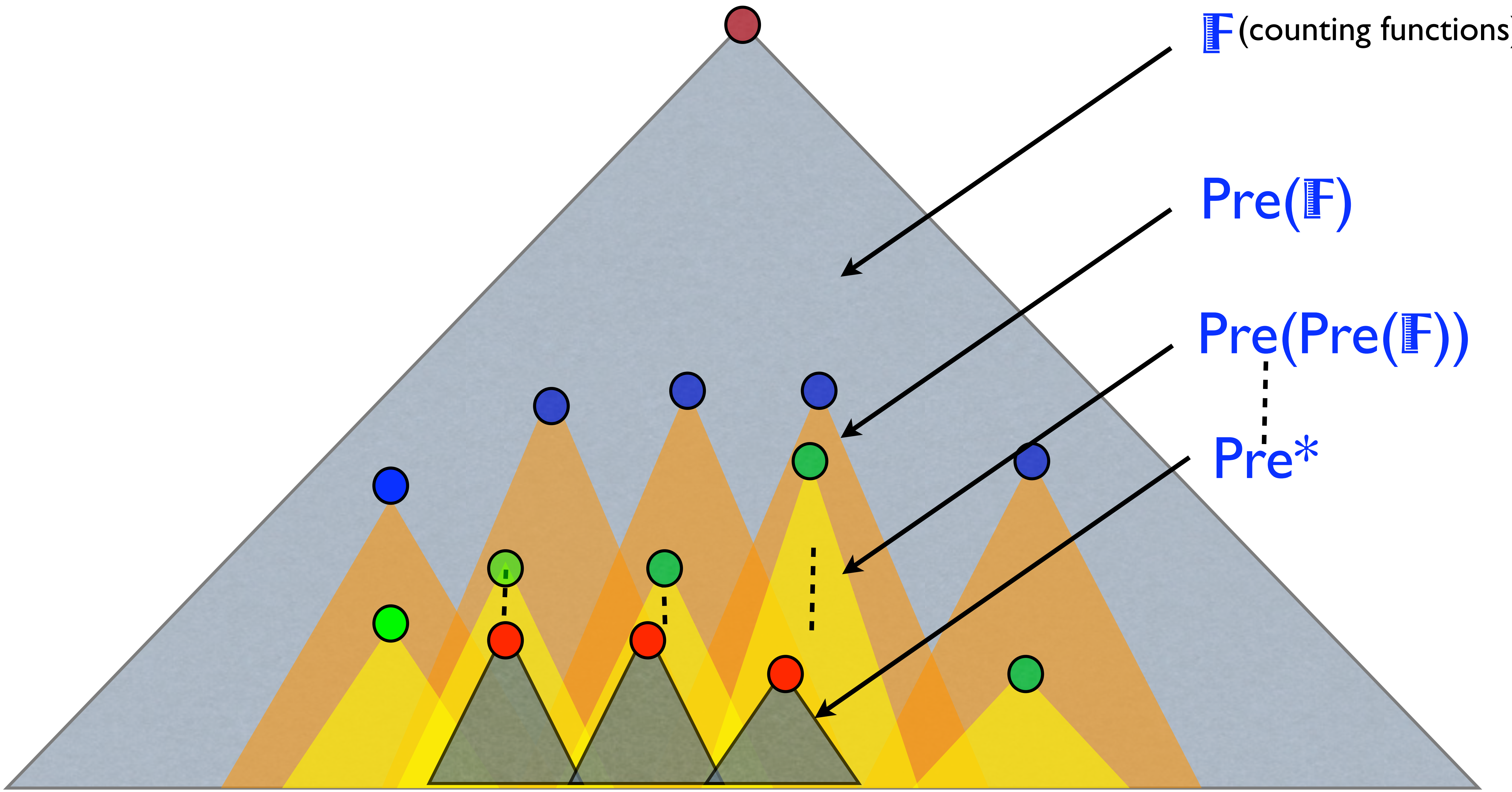

Incremental Algorithm

- the bound **K** is very big (doubly exponential)
- if the spec is realizable with a "small" bound, it is realizable with a "big" bound
- iterate over $k=0,1,\ldots,$**K**

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Not reasonable for unrealizable specifications

Incremental Algorithm

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- if t
  rea
- ite

Not

ions

But by Martin's determination theorem:

ϕ is unrealizable for the System iff $\neg\phi$ is realizable for the Environment.

Experiments

- implementation in Perl (as Lily)
- if the spec is realizable, output a Moore machine that realizes it
- formula to automata construction borrowed from Lily (based on Wring [Somenzi, Bloem])
- significantly faster on all realizable Lily's examples
- bottleneck: formula to automaton construction

Example

```
assume StoB_REQ1=0;

assume G((StoB_REQ0=1 * BtoS_ACK0=0) -> X(StoB_REQ0=1));
assume G((StoB_REQ1=1 * BtoS_ACK1=0) -> X(StoB_REQ1=1));

assume G(BtoS_ACK0=1 -> X(StoB_REQ0=0));
assume G(BtoS_ACK1=1 -> X(StoB_REQ1=0));


(BtoS_ACK0=0 *
 G( (StoB_REQ0=0 * X(StoB_REQ0=1)) -> X(BtoS_ACK0=0 * X(F(BtoS_ACK0=1))) ) *
 G( (BtoS_ACK0=0 * X(StoB_REQ0=0)) -> X(BtoS_ACK0=0) ) *
 G(BtoS_ACK0=0 + BtoS_ACK1=0)
);


(BtoS_ACK1=0 *
 G( (StoB_REQ1=0 * X(StoB_REQ1=1)) -> X(BtoS_ACK1=0 * X(F(BtoS_ACK1=1))) ) *
 G( (BtoS_ACK1=0 * X(StoB_REQ1=0)) -> X(BtoS_ACK1=0) ) *
 G(BtoS_ACK0=0 + BtoS_ACK1=0)
);

assume RtoB_ACK0=0;
assume RtoB_ACK1=0;

assume G(BtoR_REQ0=0 -> X(RtoB_ACK0=0));
#assume G((BtoR_REQ0=1 * RtoB_ACK0=1) -> X(RtoB_ACK0=1));

assume G(BtoR_REQ1=0 -> X(RtoB_ACK1=0));
#assume G((BtoR_REQ1=1 * RtoB_ACK1=1) -> X(RtoB_ACK1=1));


assume G(BtoR_REQ0=1 -> X(F(RtoB_ACK0=1)));
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(BtoR_REQ0=0 *
 G(RtoB_ACK0=1 -> X(BtoR_REQ0=0)) *
 G((BtoR_REQ0=1 * RtoB_ACK0=0) -> X(BtoR_REQ0=1)) *
 G((BtoR_REQ0=1 * X(BtoR_REQ0=0)) -> X( BtoR_REQ0=0 U (BtoR_REQ0=0 * BtoR_REQ1=1))) *
 G(F(BtoR_REQ0=1)) *
 G((BtoR_REQ0=0) + (BtoR_REQ1=0))
);

(BtoR_REQ1=0 *
 G(RtoB_ACK1=1 -> X(BtoR_REQ1=0)) *
 G((BtoR_REQ1=1 * RtoB_ACK1=0) -> X(BtoR_REQ1=1)) *
 G((BtoR_REQ1=1 * X(BtoR_REQ1=0)) -> X( BtoR_REQ1=0 U (BtoR_REQ1=0 * BtoR_REQ0=1))) *
 G(F(BtoR_REQ1=1)) *
 G((BtoR_REQ0=0) + (BtoR_REQ1=0))
);
```

```
(BtoS_ACK0=0 *
 G( (StoB_REQ0=0 * X(StoB_REQ0=1)) -> X(BtoS_ACK0=0 * X(F(BtoS_ACK0=1))) ) *
 G( (BtoS_ACK0=0 * X(StoB_REQ0=0)) -> X(BtoS_ACK0=0) ) *
 G(BtoS_ACK0=0 + BtoS_ACK1=0)
);


(BtoS_ACK1=0 *
 G( (StoB_REQ1=0 * X(StoB_REQ1=1)) -> X(BtoS_ACK1=0 * X(F(BtoS_ACK1=1))) ) *
 G( (BtoS_ACK1=0 * X(StoB_REQ1=0)) -> X(BtoS_ACK1=0) ) *
 G(BtoS_ACK0=0 + BtoS_ACK1=0)
);



(BtoR_REQ0=0 *
 G(RtoB_ACK0=1 -> X(BtoR_REQ0=0)) *
 G((BtoR_REQ0=1 * RtoB_ACK0=0) -> X(BtoR_REQ0=1)) *
 G((BtoR_REQ0=1 * X(BtoR_REQ0=0)) -> X( BtoR_REQ0=0 U (BtoR_REQ0=0 * BtoR_REQ1=1))) *
 G(F(BtoR_REQ0=1)) *
 G((BtoR_REQ0=0) + (BtoR_REQ1=0))
);

(BtoR_REQ1=0 *
 G(RtoB_ACK1=1 -> X(BtoR_REQ1=0)) *
 G((BtoR_REQ1=1 * RtoB_ACK1=0) -> X(BtoR_REQ1=1)) *
 G((BtoR_REQ1=1 * X(BtoR_REQ1=0)) -> X( BtoR_REQ1=0 U (BtoR_REQ1=0 * BtoR_REQ0=1))) *
 G(F(BtoR_REQ1=1)) *
 G((BtoR_REQ1=0) + (BtoR_REQ1=1))
);
```

Future Work ...

- compositionnality
- avoid automata construction to handle larger formulas

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... Thank You