Transducer Theory and Streaming Transformations

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Finite State Automata

- finite string acceptors over a finite alphabet Σ
- read-only input tape, left-to-right
- finite set of states

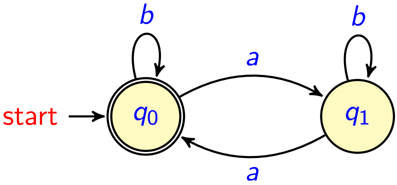
Definition (Finite State Automaton)

A finite state automaton (FA) on Σ is a tuple $A = (Q, I, F, \delta)$ where

- Q is the set of states,
- $I \subseteq Q$, reps. $F \subseteq Q$ is the set of initial, resp. final, states,
- $\delta : Q \times \Sigma \to Q$ is the transition relation.

$$L(A) \ = \ \{w \in \Sigma^* \mid \text{there exists an accepting run on } w\}$$

Finite State Automata – Example



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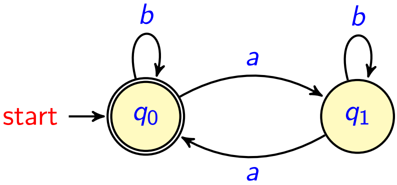

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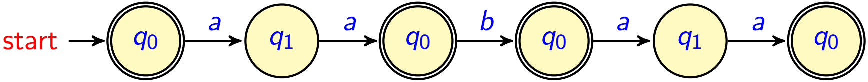

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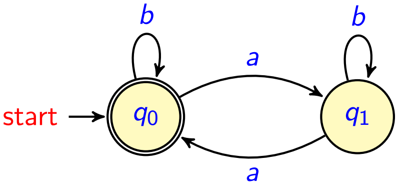

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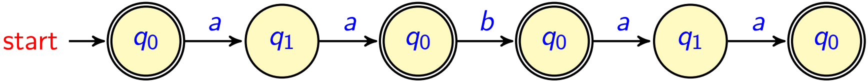

$$L(A) = \{w \in \Sigma^* \mid w \text{ contains an even number of } a\}$$

Properties of FA

Expressiveness

FA = regular languages = MSO[+1] = regular expressions = ...

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Closure Properties

- closed under Boolean operations (union, intersection, complement).
- closed under various extensions:
  - non-determinism (NFA): $\delta \subseteq Q \times \Sigma \times Q$
  - two-way input head (2NFA): $\delta \subseteq Q \times \Sigma \times \{-1, 0, 1\} \times Q$
  - regular look-ahead: $\delta \subseteq Q \times \Sigma \times \mathit{Reg} \times Q$
  - alternation: $\delta : Q \times \Sigma \to B(Q)$ (Boolean formulas over Q)

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Decision Problems

Membership, emptiness, universality, inclusion, equivalence ... are decidable.

From Languages to Transductions

Let Σ and Δ be two finite alphabets.

Definition

Language on Σ	Transduction from Σ to Δ
function from Σ^* to $\{0,1\}$	relation $R \subseteq \Sigma^* \times \Delta^*$
defined by automata	defined by transducers
accept strings	transform strings

transducer $=$ <u>automaton</u> $+$ output mechanism.

Finite State Transducers

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- read-only left-to-right input head
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Definition (Finite State Transducers)

A finite state transducer from Σ to Δ is a pair $T = (A, O)$ where

- $A = (Q, I, F, \delta)$ is the underlying automaton
- O is an output morphism from δ to Δ^*.

- If $t = q \xrightarrow{a} q' \in \delta$, then $O(t)$ defines its output.
- $q \xrightarrow{a|w} q'$ denotes a transition whose output is $w \in \Delta^*$.

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Two classes of transducers:

- DFT if A is deterministic
- NFT if A is non-deterministic.

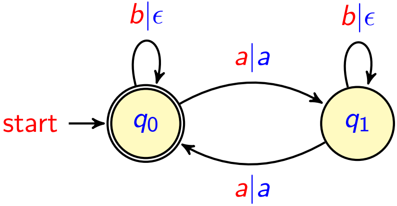
Some applications

- language and speech processing (e.g. see work by Mehryar Mohri)
- model-checking infinite state-space systems[1]
- verification of web sanitizers[2]
- string pattern matching

[1] A survey of regular model checking, P. Abdulla, B. Jonsson, M. Nilsson, M. Saksena. 2004

[2] see BEK, developped at Microsoft Research

Finite State Transducers – Example 1



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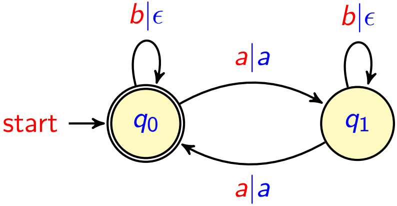

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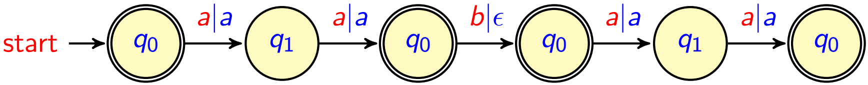

$$T(aabaa) = a.a.\epsilon.a.a = aaaa.$$

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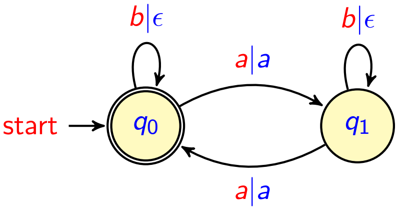

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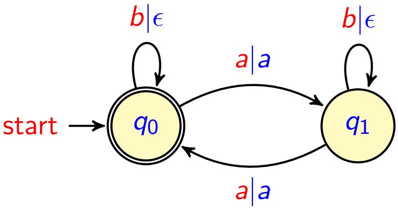

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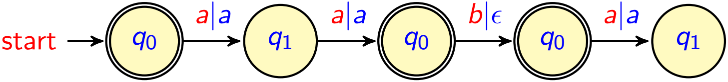

$T(aaba) =$ **undefined**

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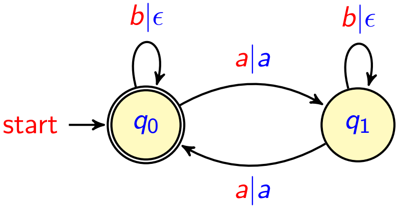
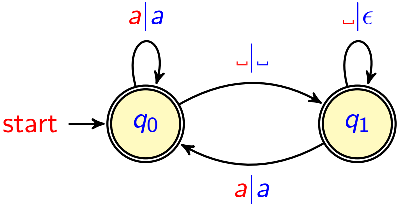


Semantics

$$dom(T) = \{w \in \Sigma^* \mid \#_a w \text{ is even}\}$$
$$R(T) = \{(w, a^{\#_a w}) \mid w \in dom(T)\}$$

Finite State Transducers – Example 2

␣ = white space



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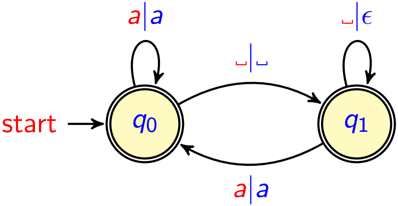

Semantics

Replace blocks of consecutive white spaces by a single white space.

$$T(\text{␣␣}aa\text{␣␣␣}a\text{␣␣}) = \text{␣}aa\text{␣}a\text{␣}$$

Finite State Transducers – Example 3

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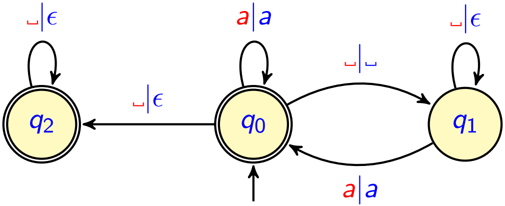
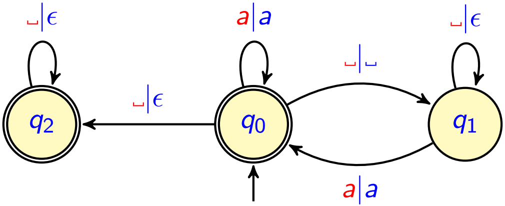


Finite State Transducers – Example 3

$\llcorner\!\lrcorner$ = white space



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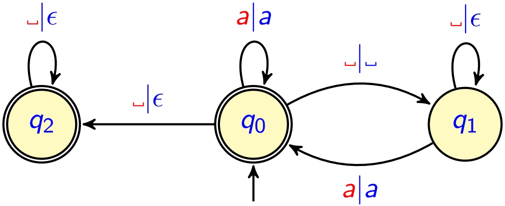

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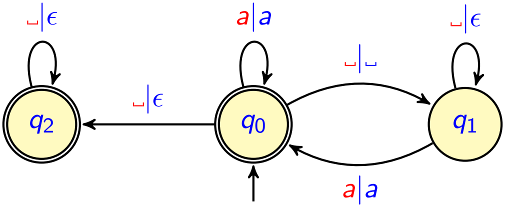
Non-deterministic but still defines a function: functional NFT

Is non-determinism needed ?



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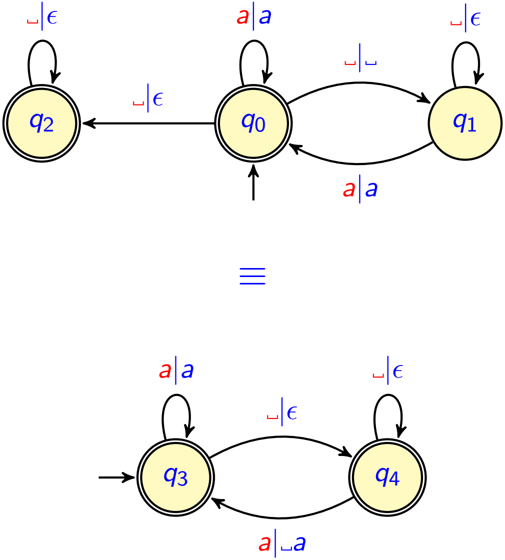

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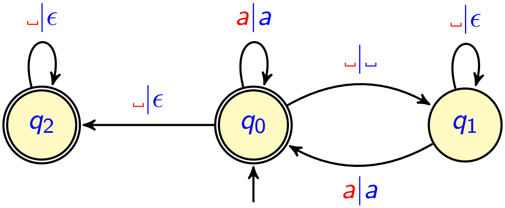

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- output the longest common prefix

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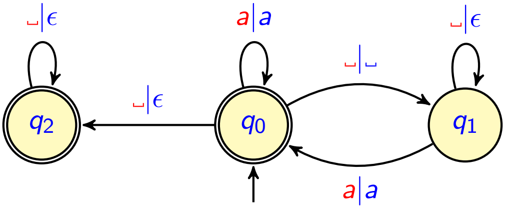

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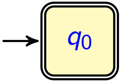

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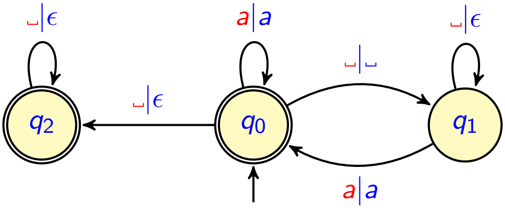

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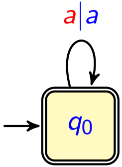

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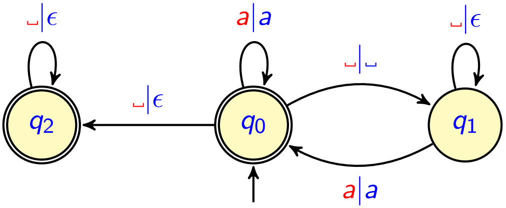

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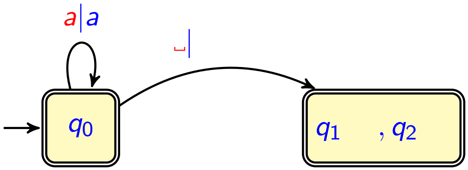

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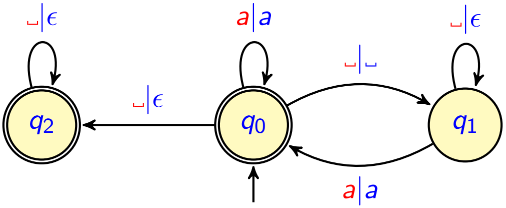

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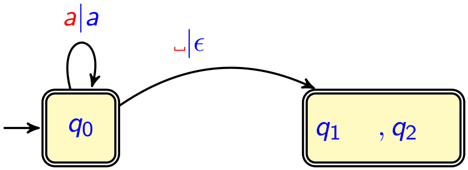

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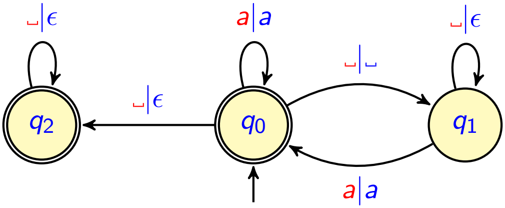

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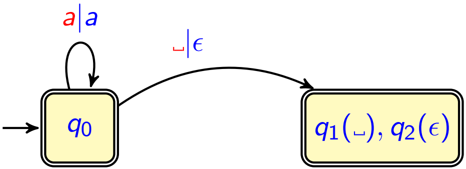

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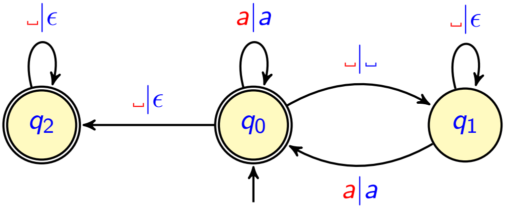

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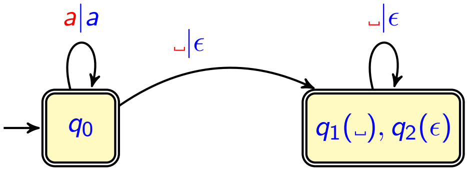

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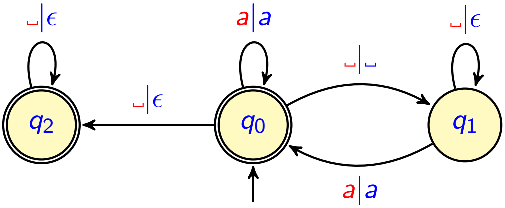

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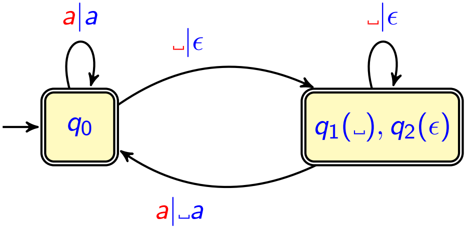

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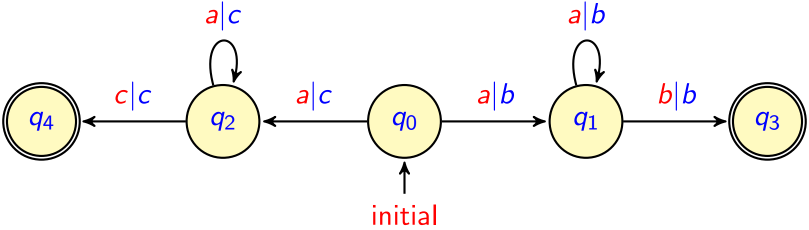
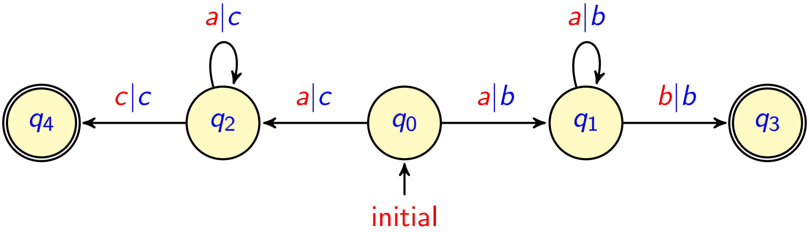


Semantics

$$R(T): \begin{cases} a^n b \mapsto b^{n+1} \\ a^n c \mapsto c^{n+1} \end{cases}$$

functional but not determinizable

Subset construction fails ...



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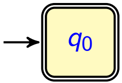

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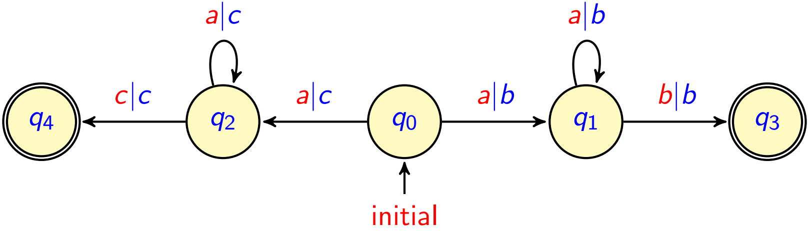

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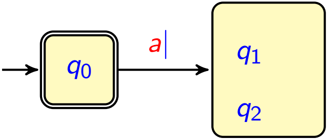

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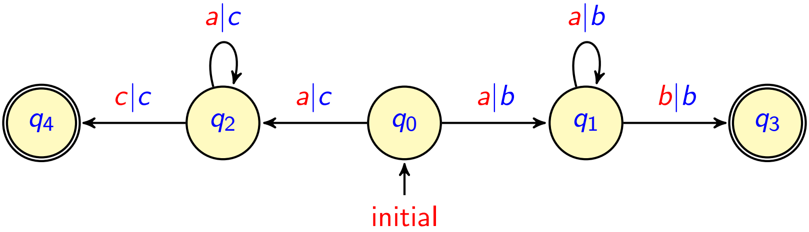

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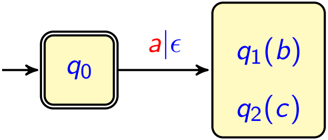

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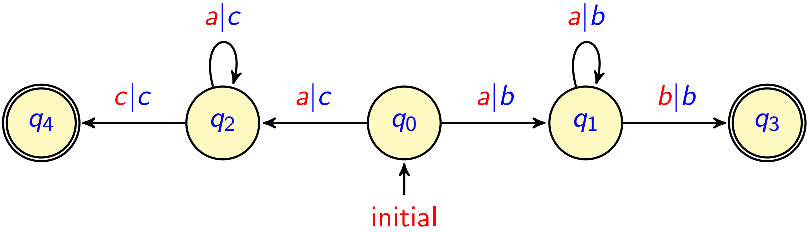

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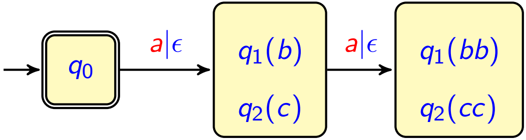

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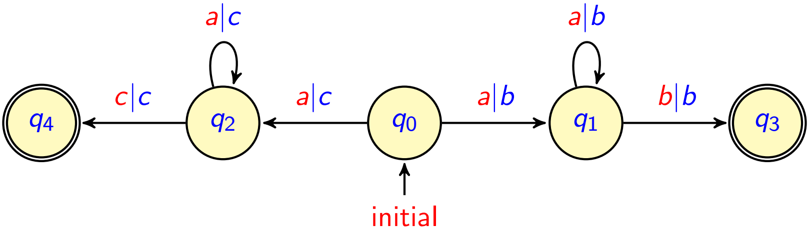

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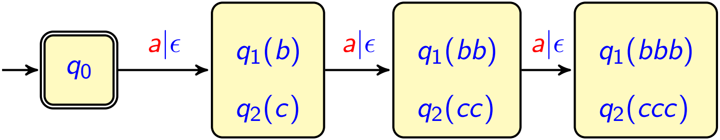

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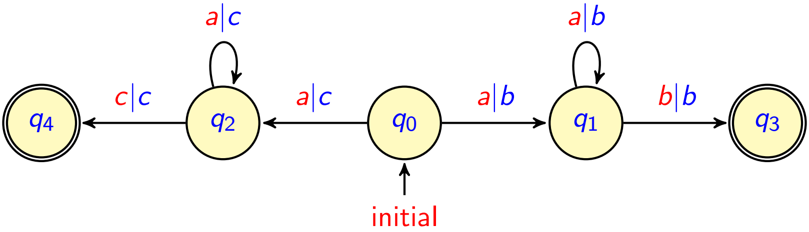

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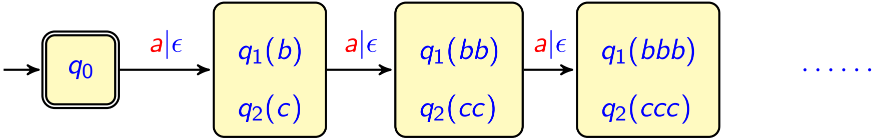

How to guarantee termination of subset construction?

LAG

$LAG(u, v) = (u', v')$ such that $u = \ell u'$, $v = \ell v'$ and $\ell = lcp(u, v)$.

E.g. $LAG(abbc, abc) = (bc, c)$.

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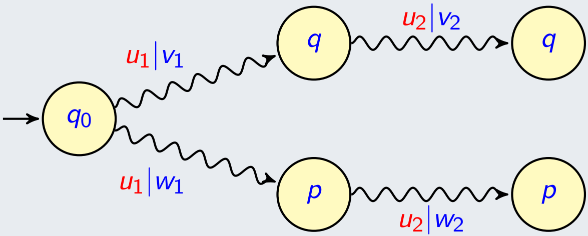
Lemma (Twinning Property)

Subset construction terminates **iff** *for all such situations*



it is the case that $LAG(v_1, w_1) = LAG(v_1 v_2, w_1 w_2)$.

Determinizability is decidable

Theorem (Choffrut 77, Beal Carton Prieur Sakarovitch 03)

Given a functional NFT T, the following are equivalent:

1. *it is determinizable*
2. *the twinning property holds.*

Moreover, the twinning property is decidable in PTime.

Proof.

Intuition

- If TP holds, then subset construction terminates and produces an equivalent DFT
- for the converse, uses the fact that TP is machine-independent: for all $T \equiv T'$, $T \models TP$ iff $T' \models TP$.

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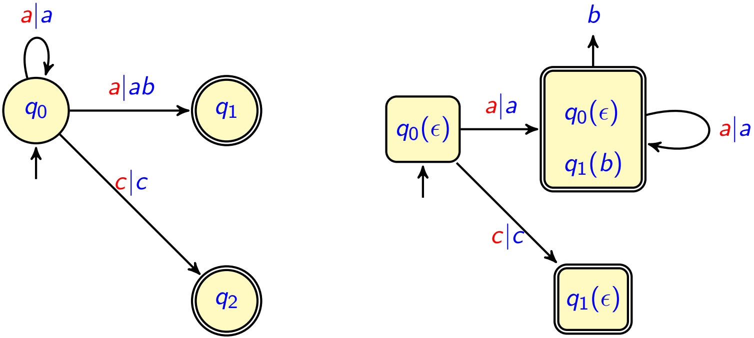
Almost true ...

True if ...



- subsequential transducers are deterministic but can output a string in each accepting states
- in the previous theorem: "determinizable" $\leftrightarrow$ "there exists an equivalent subsequential transducer"
- subsequential transducers $\equiv$ DFT if last string symbol is unique

Application: analysis of streaming transformations

Bounded Memory Problem

Hypothesis:

- input string is received as a (very long) **stream**
- output string is produced as a stream

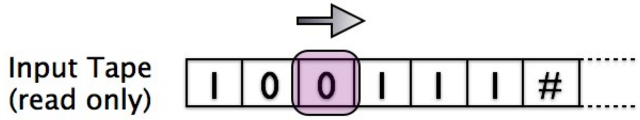
Input: a transformation defined by some functional NFT
Output: can I realize this transformation with bounded memory ?

$$\exists B \in \mathbb{N} \cdot \forall u \in dom(T)$$

$T(u)$ can be computed with B-bounded memory ?

Streaming Model

Deterministic Turing Transducer



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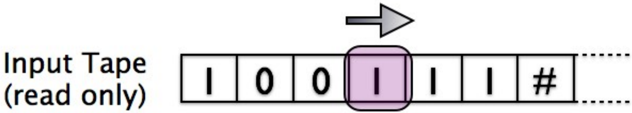

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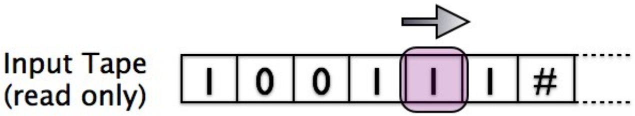

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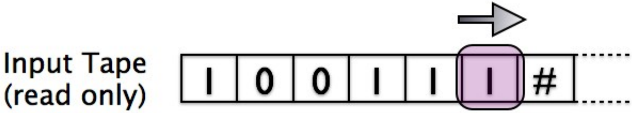

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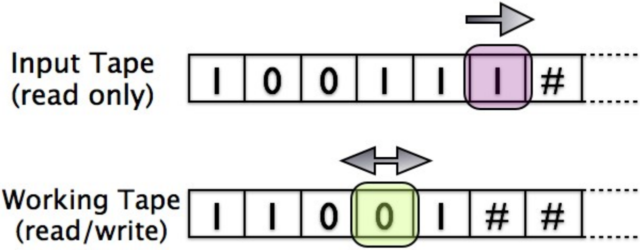

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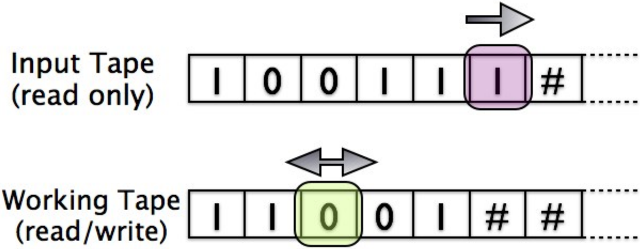

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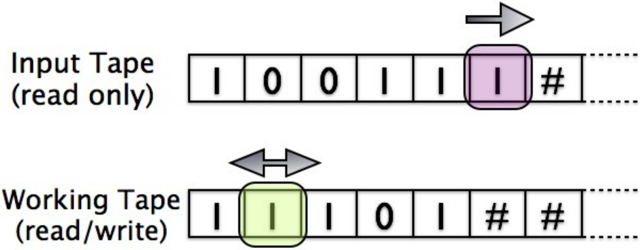

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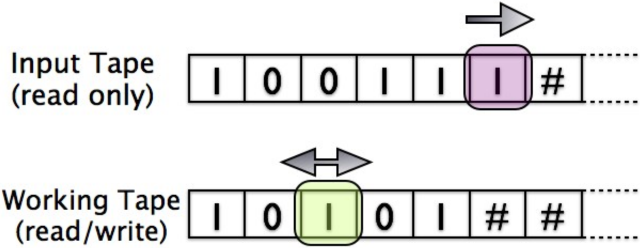

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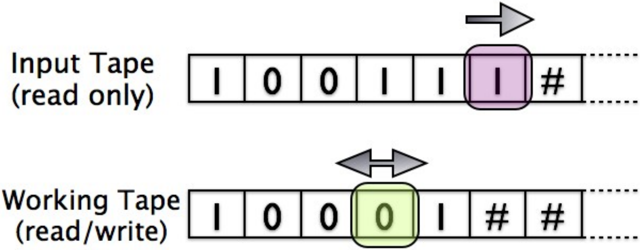

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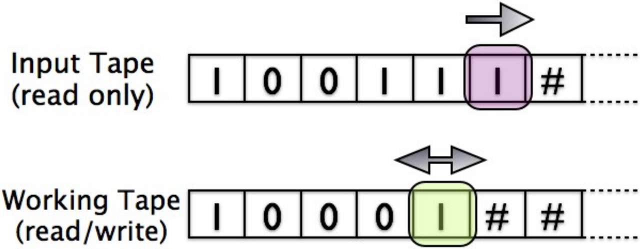

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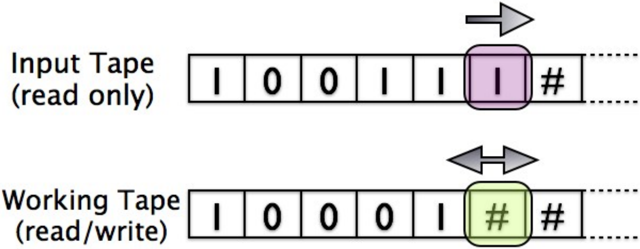

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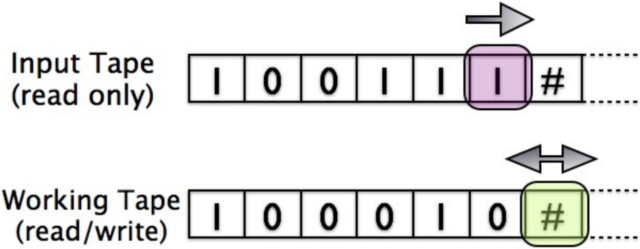

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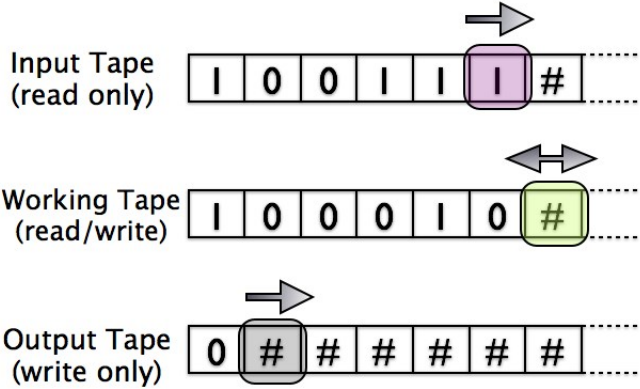

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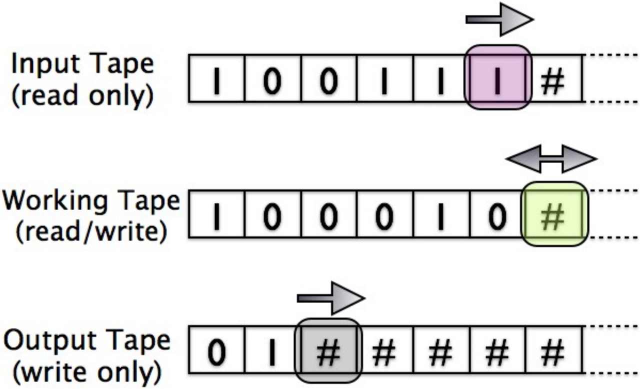

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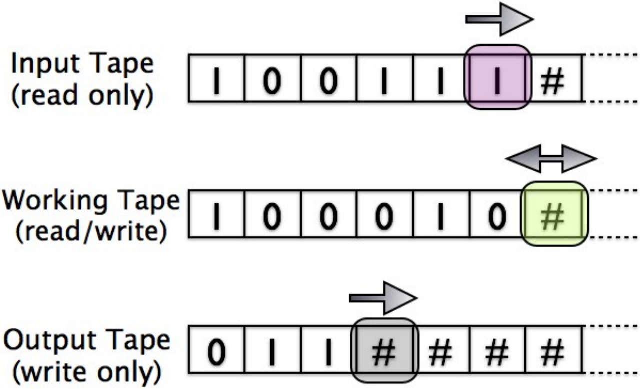

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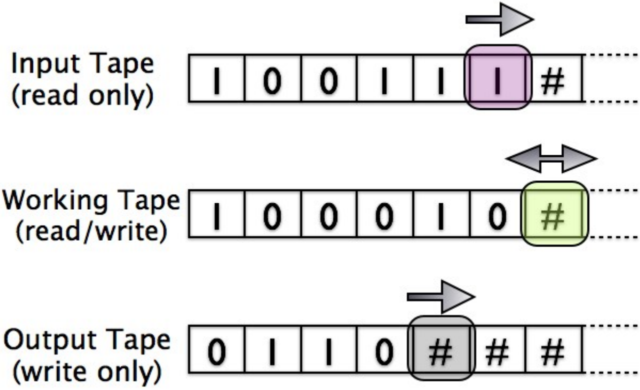

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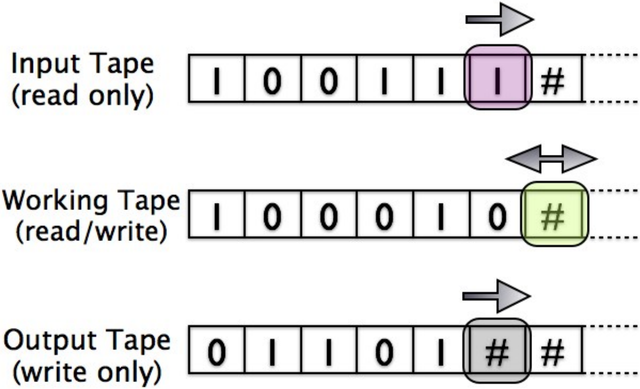

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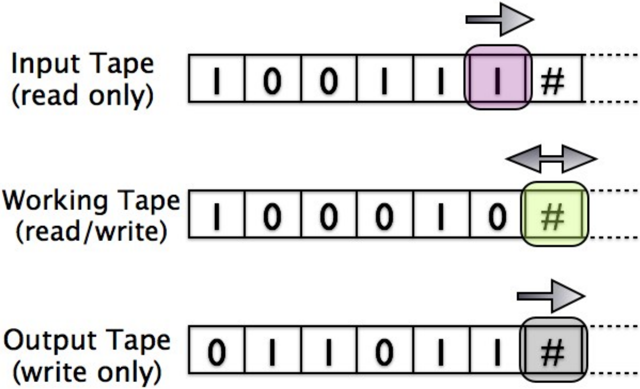

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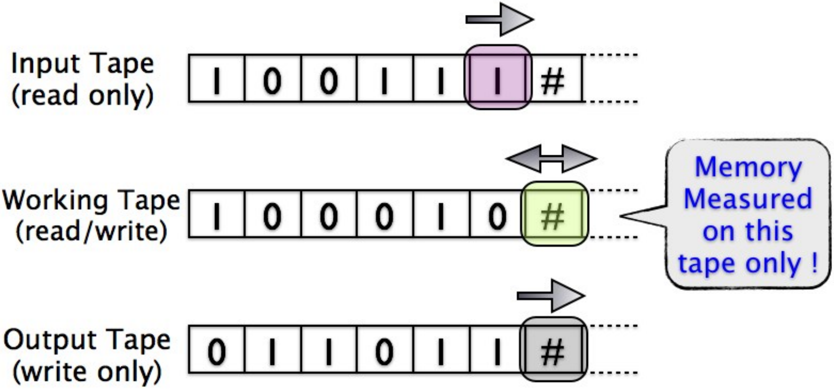

Bounded Memory Problem – Examples

$T_1 : \begin{cases} a^n b \mapsto b^{n+1} \\ a^n c \mapsto c^{n+1} \end{cases}$	Not bounded memory
$T_2 : \textvisiblespace\textvisiblespace a \textvisiblespace\textvisiblespace\textvisiblespace b \textvisiblespace\textvisiblespace \mapsto \textvisiblespace a \textvisiblespace b$	Bounded memory

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Theorem

For all functional NFT T, the following are equivalent:

1. *T is bounded memory*
2. *T is determinizable*
3. *T satisfies the twinning property.*

Proof based on the following two observations:

1. any DFT is bounded memory
2. bounded memory Turing Transducer $\equiv$ DFT

Closure Properties of Finite State Transducers

Domain, co-domain

The domains and co-domains of NFT are regular.

	T^{-1}	$\overline{T}$	$T_1 \cup T_2$	$T_1 \cap T_2$	$T_1 \circ T_2$
NFT	no	no	yes	no	yes
DFT	no	no	no	no	yes

Table: Closure Properties for NFT and DFT.

Closure Properties of Finite State Transducers

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Table: Closure Properties for NFT and DFT.

Non-closure by intersection

1. $R(T_1) = \{(a^m b^n, c^m) \mid m, n \geq 0\}$
2. $R(T_2) = \{(a^m b^n, c^n) \mid m, n \geq 0\}$
3. $R(T_1) \cap R(T_2) = \{(a^n b^n, c^n) \mid n \geq 0\}$

Decision problems

Membership $(u, v) \in R(T)$?

Emptiness $R(T) = \emptyset$?

Type checking $T(L_{in}) \subseteq L_{out}$?

Equivalence $R(T_1) = R(T_2)$?

Inclusion $R(T_1) \subseteq R(T_2)$?

	emptiness / membership	type checking (vs NFA)	equiv / inclusion
NFT	PTIME	PSPACE-C	undec
DFT	PTIME	PSPACE-C	PTIME

Table: Decision problems for NFT and DFT.

Undecidability of equivalence and inclusion proved in [Griffiths68].

Functional Finite State Transducers

A transduction (transducer) is functional if each word has at most 1 image.

Theorem (Gurari and Ibarra 83)

Functionality is decidable in PTIME *for* NFT.

Theorem

The equivalence and inclusion of functional NFT is PSPACE-c.

Proof.

T_1 is included in T_2 if and only if

- $dom(T_1) \subseteq dom(T_2)$, and
- $T_1 \cup T_2$ is functional.

$\square$

k-valued Finite State Transducers

A transduction (transducer) is k-valued if each word has at most k images.

Theorem (GI83, Web89, SdS08)

Let $k \in \mathbb{N}$ be fixed.
k-valuedness is decidable in $\mathrm{PT{\scriptsize IME}}$ for NFT.

Theorem (IK86, Web88)

The equivalence and inclusion of k-valued NFT are $\mathrm{PS{\scriptsize PACE}}$-$\mathrm{{\scriptsize C}}$.

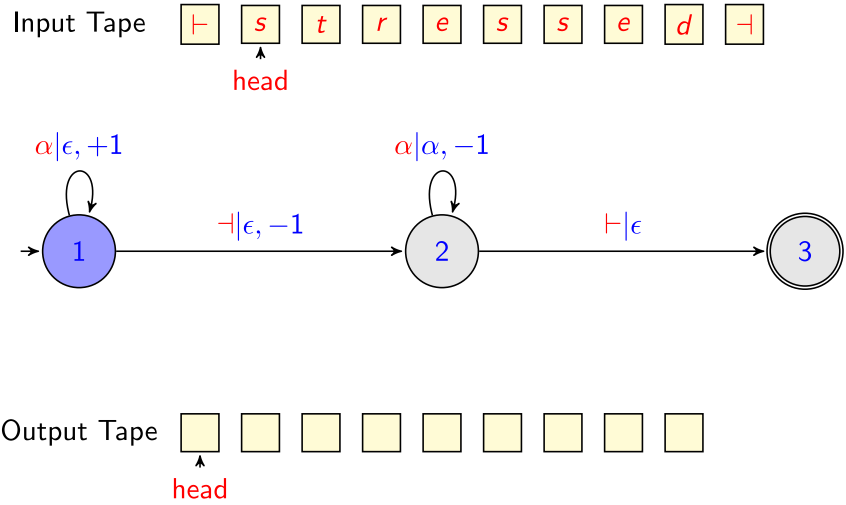
Extensions of NFT

Extensions of NFT

Various more expressive extensions have been considered:

1. two-way input tape
2. string variables (Alur Cerny 2010)
3. pushdown stack

Two-way finite state transducers (2NFT)



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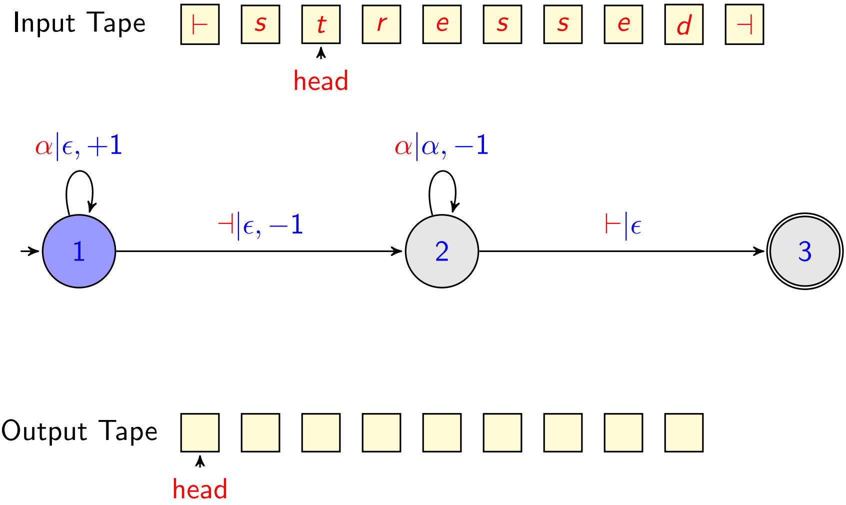

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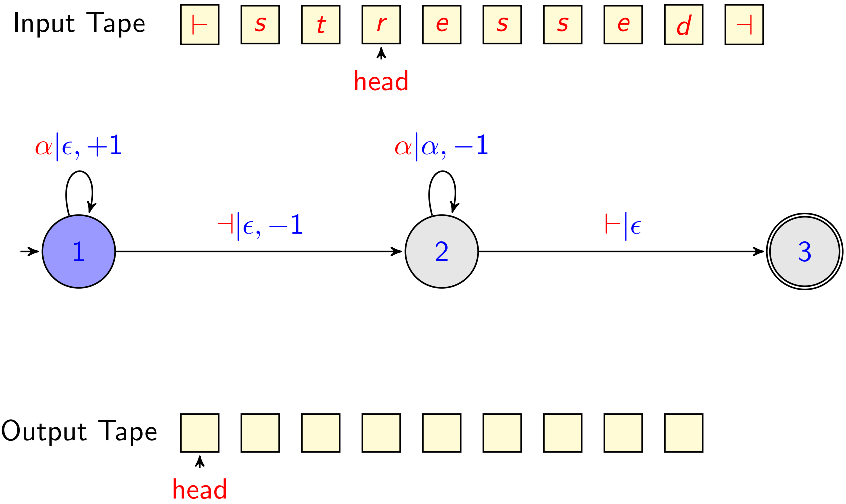

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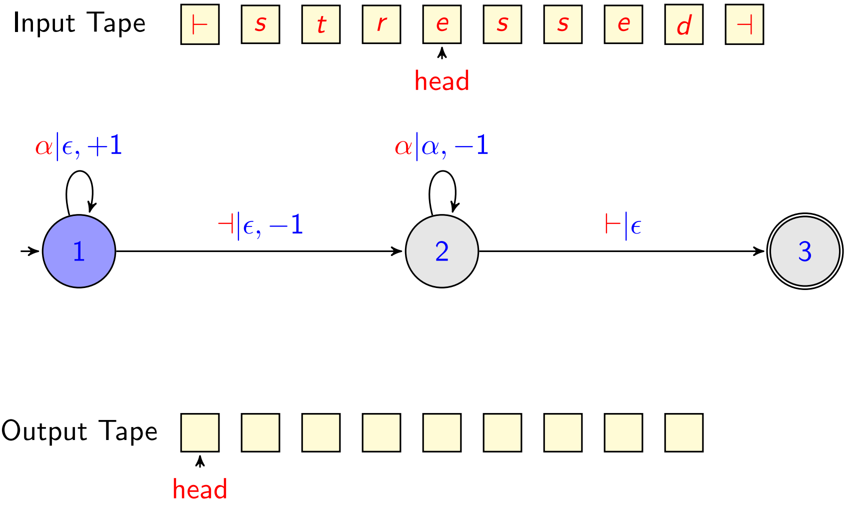

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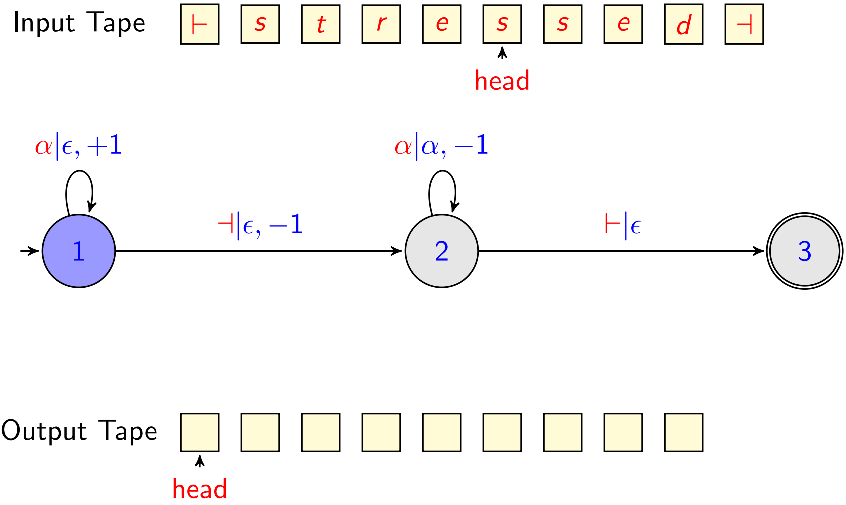

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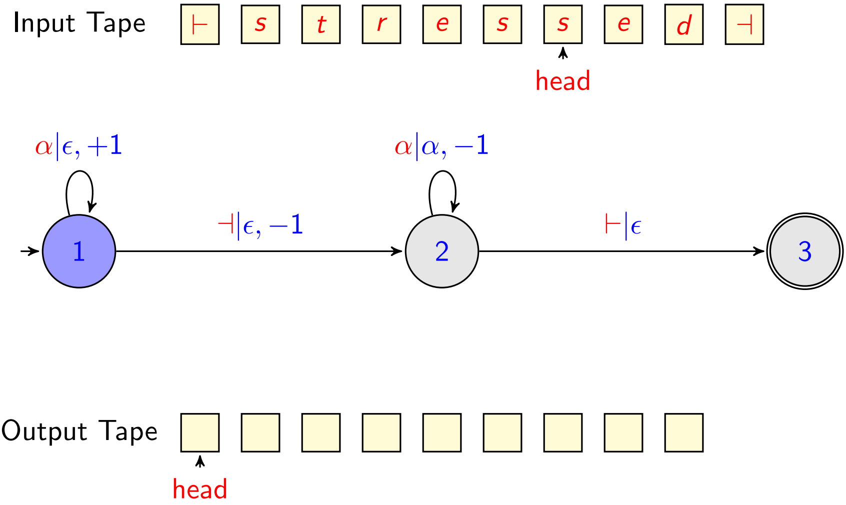

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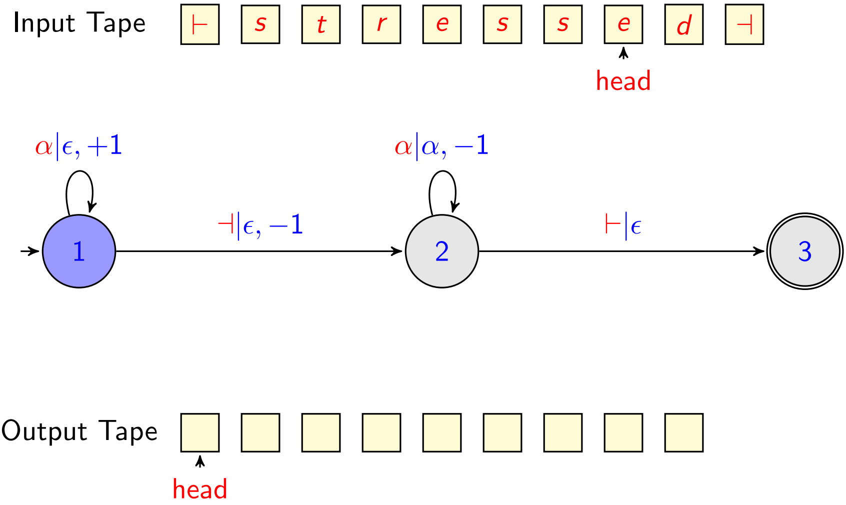

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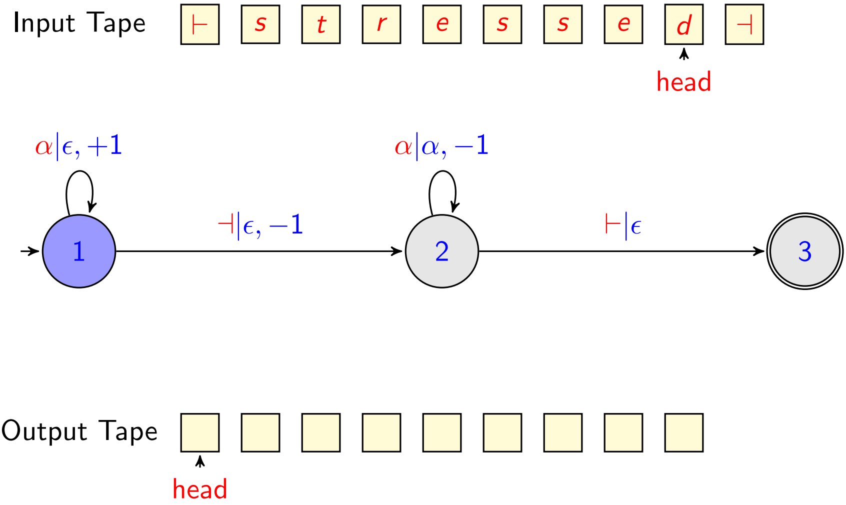

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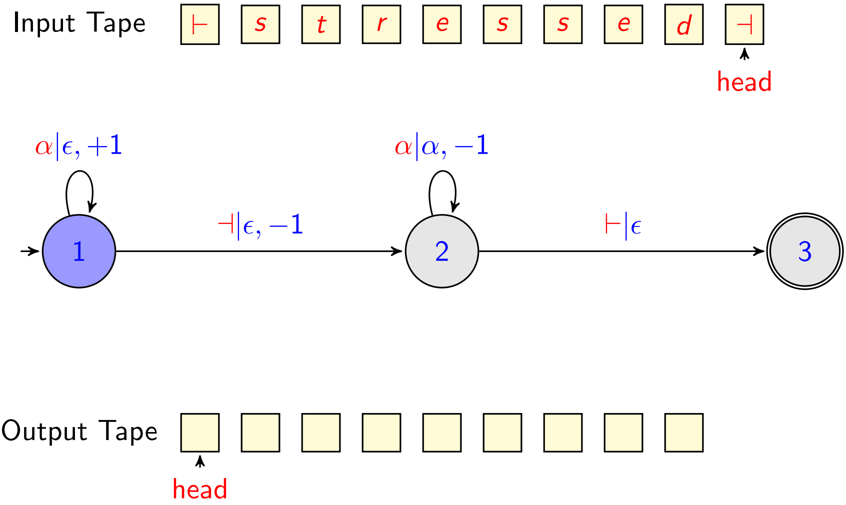

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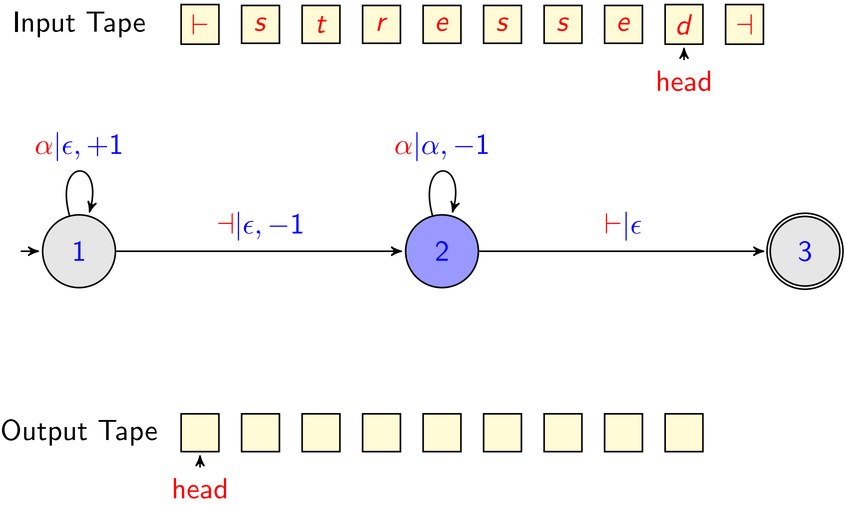

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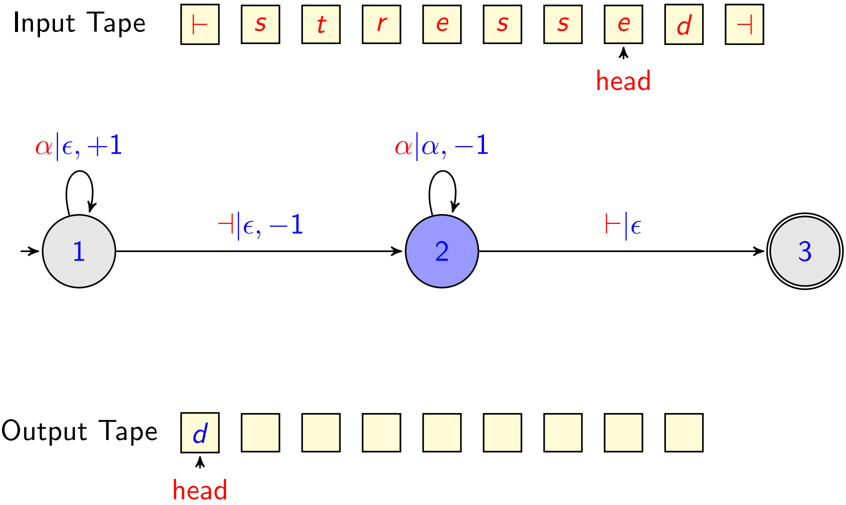

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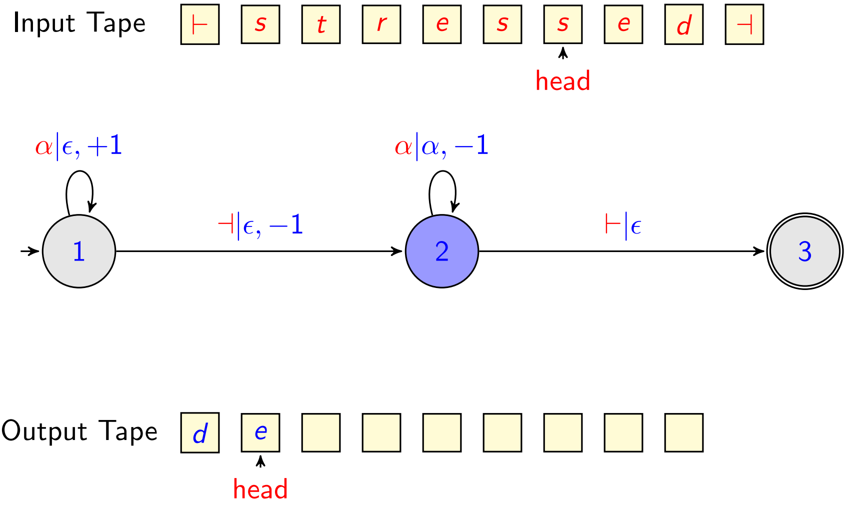

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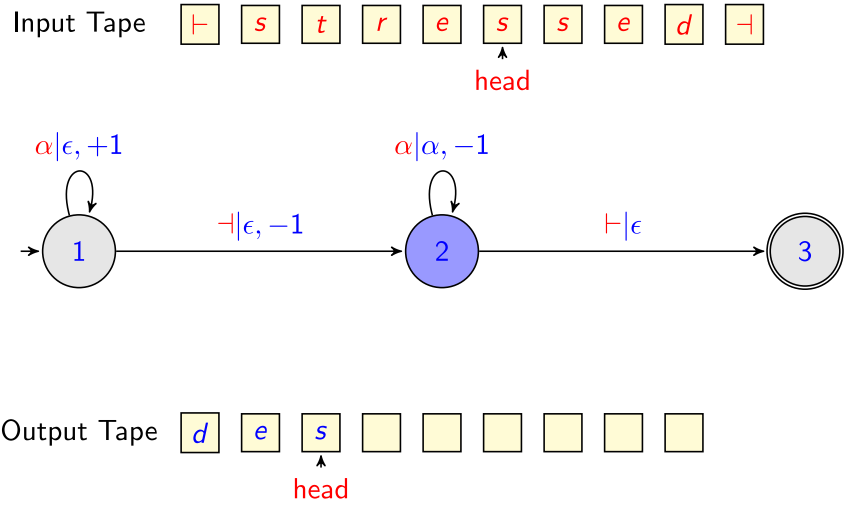

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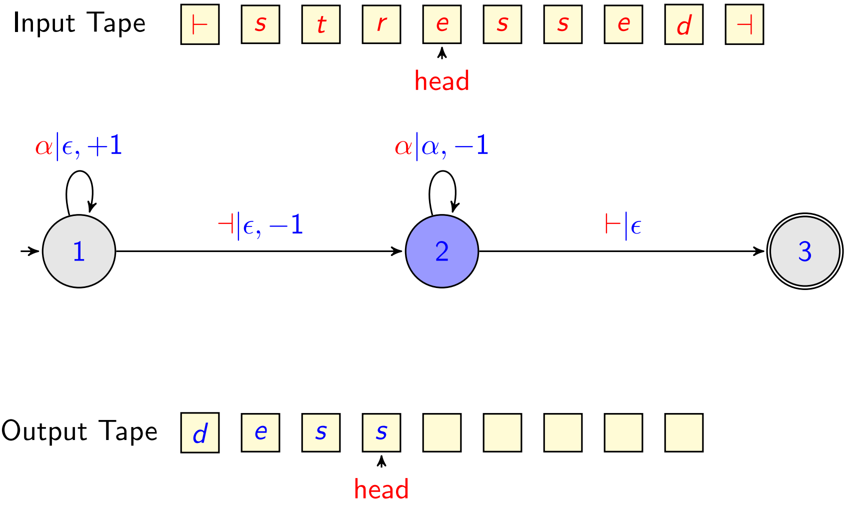

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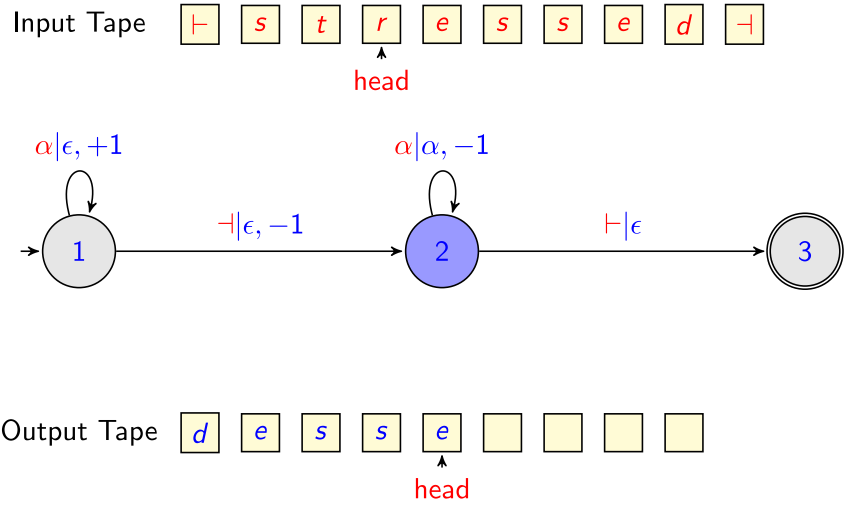

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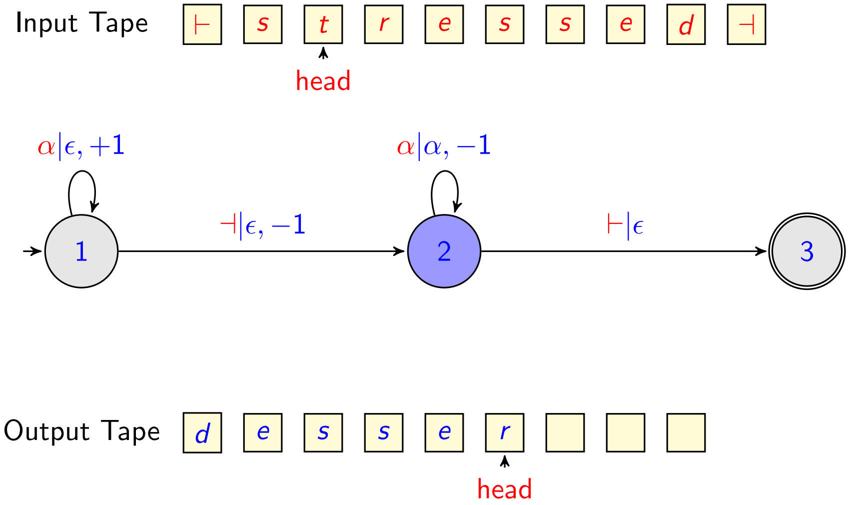

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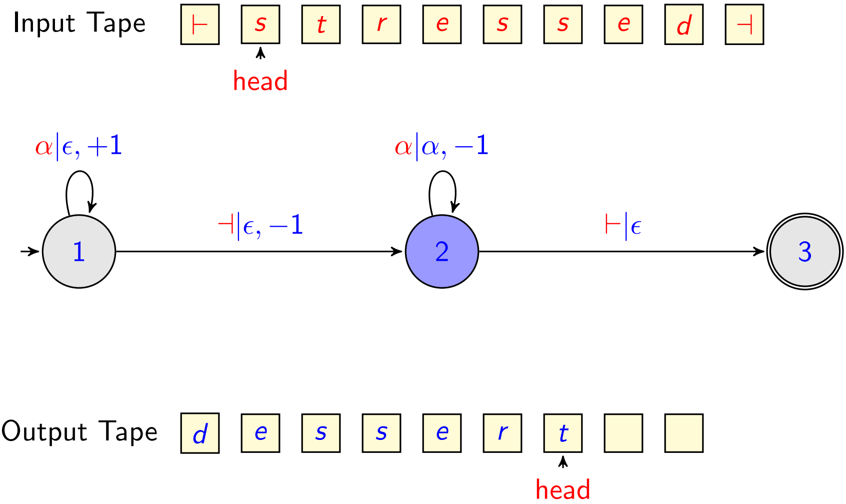

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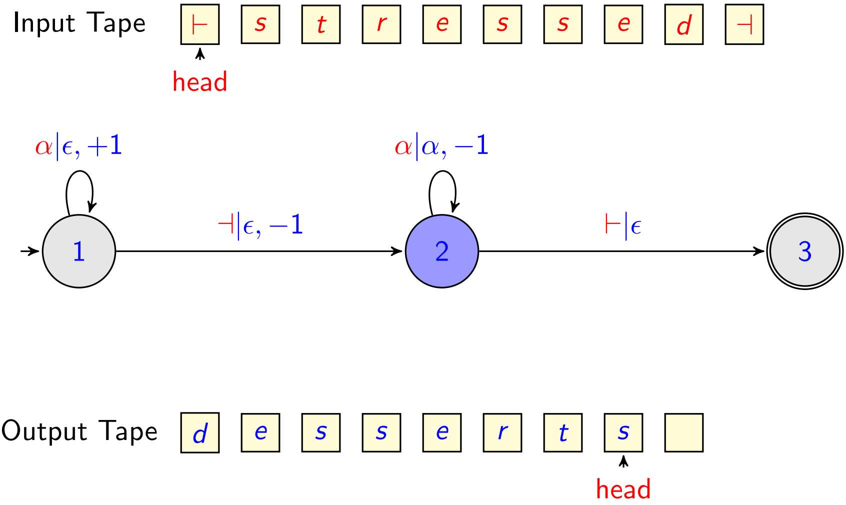

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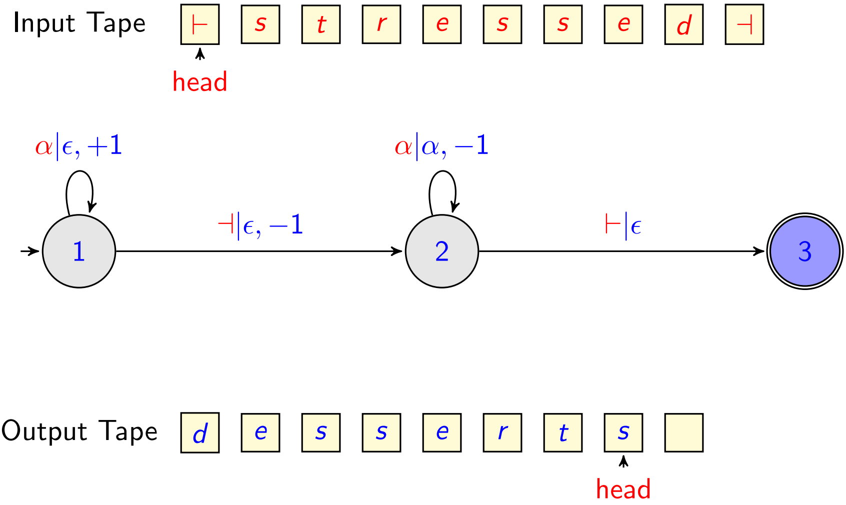

Two-way finite state transducers – Properties

Main Properties of 2NFT

1. still closed under composition (Chytil Jakl 77)
2. equivalence of functional 2NFT is decidable (Culik, Karhumaki, 87)
3. functional 2NFT $\equiv$ 2DFT (Hoogeboom Engelfriet 01, De Souza 13)

Logical Characterization (Hoogeboom Engelfriet 01)

$$\text{2DFT} \equiv \text{MSO transductions}$$

2DFT define regular functions.

MSO Transductions (Courcelle)

- input string seen as the logical structure over $\{succ, (lab_a)_{a\in\Sigma}\}$
- output predicates defined with MSO formulas interpreted over the input structure

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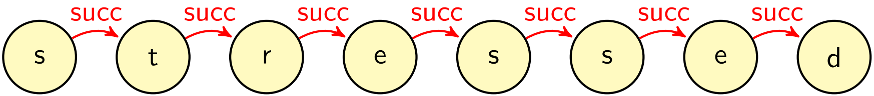

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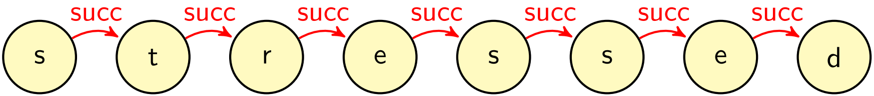

$$\phi_{succ}(x,y) \equiv succ(y,x)$$

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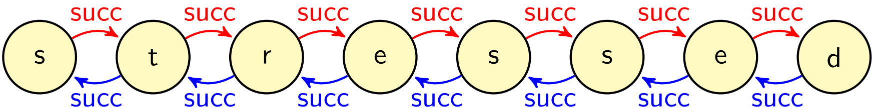

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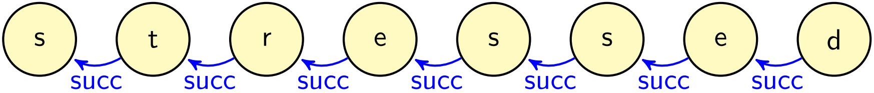

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Streaming String Transducers (Alur, Cerny, 2010)

On every transitions, a finite set of variables can be updated by

- appending a string: $x := x.u$
- prepending a string: $x := u.x$
- concatenating two variables: $x := yz$

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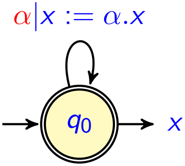

$R(T) = mirror$

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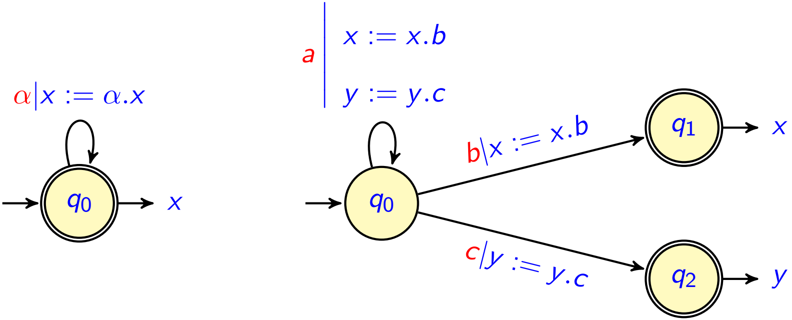

$R(T) = mirror$

$R(T) = a^n\alpha \mapsto \alpha^{n+1}$

Streaming String Transducers

Theorem (Alur Cerny 2010)

The following models are expressively equivalent:

1. *two-way DFT*
2. *MSO transductions*
3. *deterministic (one-way) streaming string transducers with copyless update*

Moreover, SSTs have good algorithmic properties and have been used to analyse list processing programs (Alur Cerny 2011).

Pushdown Transducers

Definition

A pushdown transducer is a pair (A, O) where A is a pushdown automaton and O is an output morphism.

(Bad) Properties

- closure under composition is lost
- Functionality, determinizability, equivalence and inclusion of functional transducers are lost.

Finite State Transducers – Summary

D="(input) deterministic"

f="functional"

DFTs	fNFTs	NFTs
2DFTs	f2NFTs	2NFTs

Finite State Transducers – Summary

D="(input) deterministic"

f="functional"

$\vdash u \dashv \mapsto \text{mirror}(u)$

DFTs fNFTs NFTs

$\cap$ $\cap$ $\cap$

2DFTs f2NFTs 2NFTs

Finite State Transducers – Summary

D="(input) deterministic"

f="functional"

$\{(a,a),(a,b)\}$

DFTs	fNFTs	$\subset$	NFTs
$\cap$	$\cap$		$\cap$
2DFTs	f2NFTs	$\subset$	2NFTs

Finite State Transducers – Summary

D="(input) deterministic"

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2DFTs		f2NFTs	$\subset$	2NFTs

Finite State Transducers – Summary

D="(input) deterministic"

f="functional"

$$\begin{array}{ccccc} \text{DFTs} & \subsetneq & \text{fNFTs} & \subsetneq & \text{NFTs} \\ \cap & & \cap & & \cap \\ \text{2DFTs} & \equiv & \text{f2NFTs} & \subsetneq & \text{2NFTs} \end{array}$$

[De Souza (13)]

≡MSOT [Engelfriet,Hoogeboom (01)]

≡ Streaming String Transducers [Alur, Černý, 2010]

Finite State Transducers – Summary

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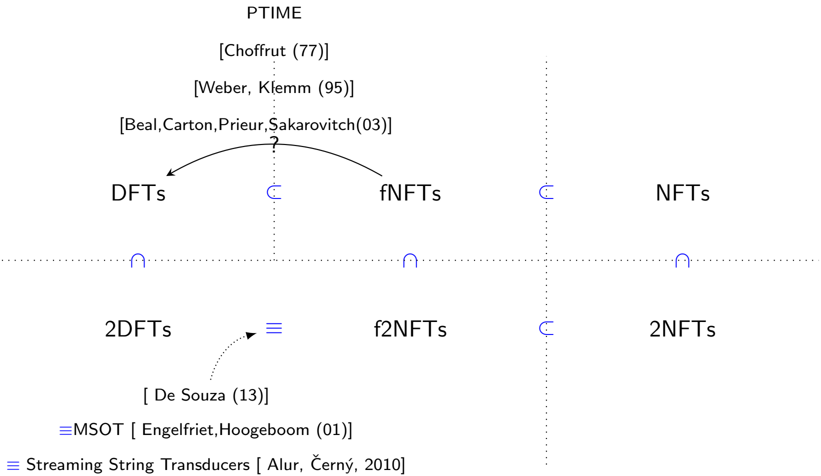

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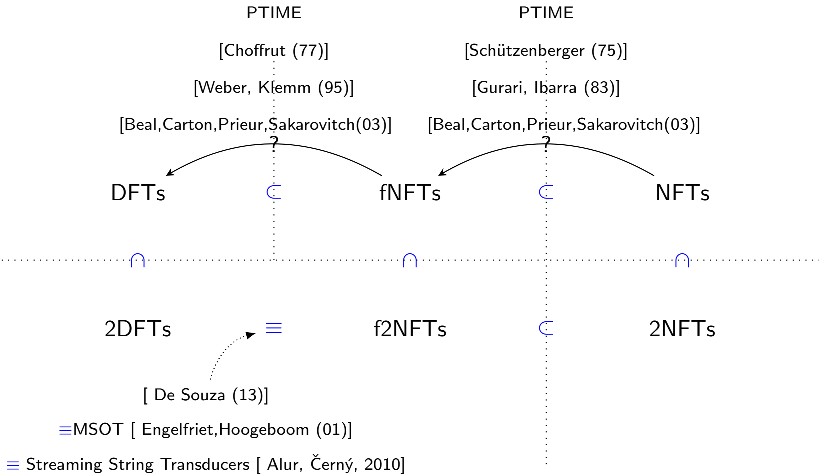

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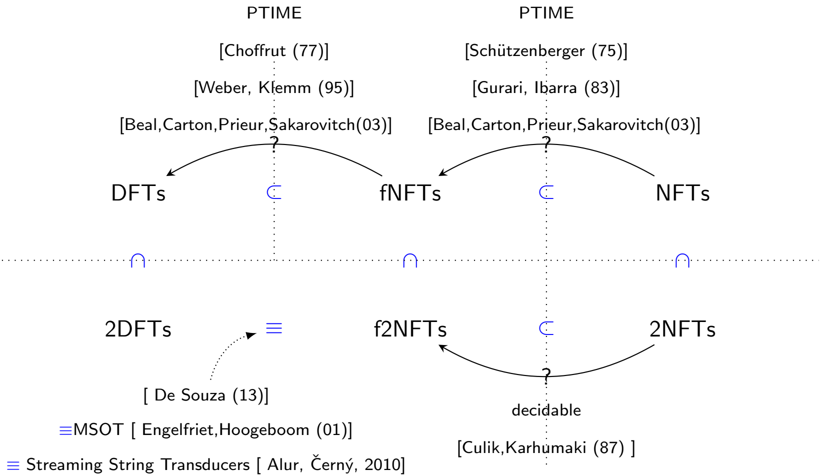

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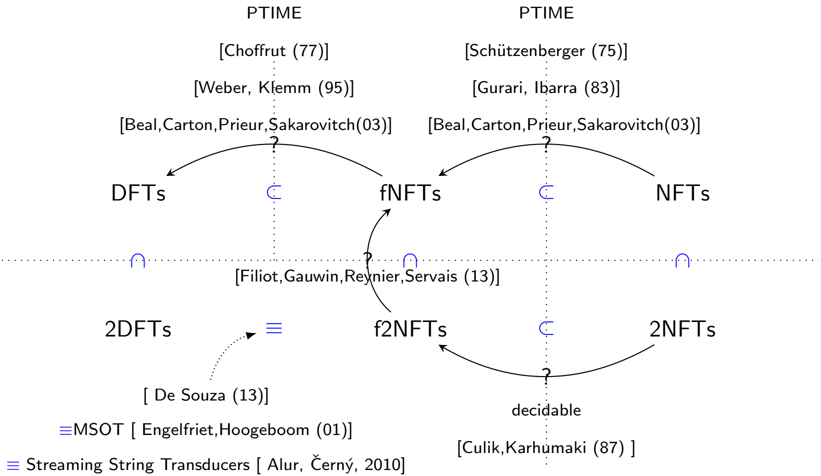
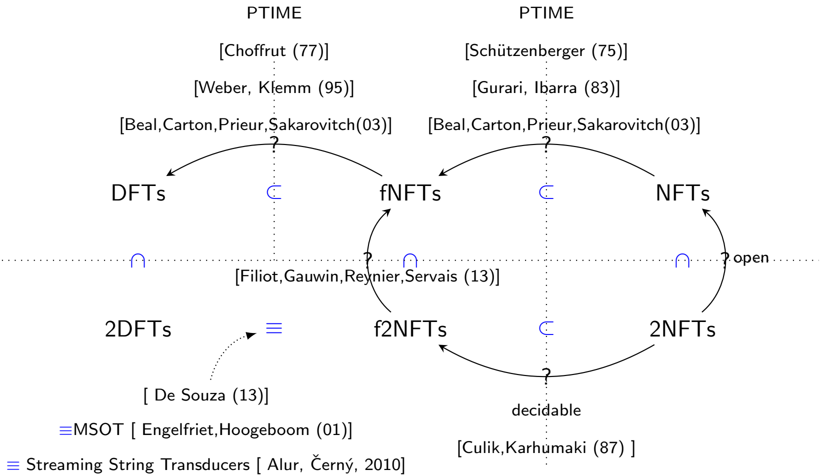


Finite State Transducers – Summary

D="(input) deterministic"

f="functional"



A word about infinite strings

- most transducer models can be extended to (right-) infinite strings
- Büchi / Muller accepting conditions
- most of the results seen so far **still hold with some complications ...**

- determinization of one-way transducers: TP is too strong

- deterministic 2way $<$ functional 2way:

$$T \; : \; u \mapsto \begin{cases} a^\omega \text{ if infinite number of 'a'} \\ u \text{ otherwise} \end{cases}$$

- functional 2way $\equiv$ determinitic 2way $+$ ω-regular look-ahead $\equiv$ ω-MSO transductions $\equiv$ ω-SST (Alur,Filiot,Trivedi,12)

Transducers for Nested Words ($\sim$ Trees)

Motivations

Streaming XML Transformations

- XML are words with a nesting structure
- XML documents can be (very) wide but usually not deep
- in a streaming setting, not reasonable to keep the entire document in memory
- bounded memory streaming transformations ?

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Visibly Pushdown Transducers (VPTs)

- extend Visibly Pushdown Automata (Alur Madhusudan 04)
- well-suited for streaming nested words transformations
- bounded memory analysis for VPT transductions.

Structured Alphabet

Definition (Structured Alphabet)

A structured alphabet, Σ, is a set $\Sigma = \Sigma_c \uplus \Sigma_i \uplus \Sigma_r$, where

- Σ_c are call symbols,
- Σ_i are internal symbols,
- Σ_r, are return symbols.

- a nested word is a word over a structured alphabet

$$c_1 \; c_2 \; a \; r_1$$

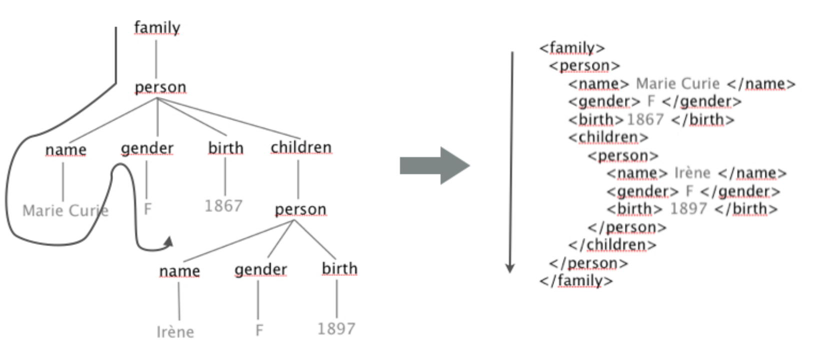
- it is well-nested if there is no pending call nor return symbols

$$c_1 \; c_2 \; a \; r_2 \; b \; r_1$$

Nested Words vs Trees

Encoding

Well-nested words $\equiv$ linearizations of trees



- nested words are well-suited to model tree streams

Visibly Pushdown Automata (VPAs) [Alur,Madhusudan,04]

VPAs = Pushdown Automata on structured alphabet
$\Sigma = \Sigma_c \uplus \Sigma_r \uplus \Sigma_i$:

- push **one** stack symbol on call symbols Σ_c
- pop **one** stack symbol on return symbols Σ_r
- don't touch the stack on internal symbols Σ_i
- in this talk, accept on empty stack and final state

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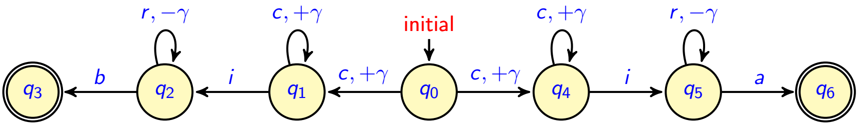


$$L(A) = \{c^n\ i\ r^n\ a \mid n > 0\} \cup \{c^n\ i\ r^n\ b \mid n > 0\}$$

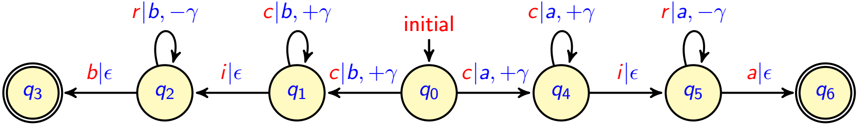
Properties of VPA

- NFA $<$ VPA $<$ PA
- close under all Boolean operations
- NFA algorithmic properties are preserved (equivalence, universality, ...)
- applicatons in
  - computer-aided verification
  - XML processing
- see `http://www.cs.uiuc.edu/~madhu/vpa/`

Visibly Pushdown Transducers (VPTs)

Definition

Pair (A, O) where $A : VPA$ and O is an output morphism.



$$R(T) = \{(c^n\ i\ r^n\ a, a^{2n}) \mid n > 0\} \cup \{(c^n\ i\ r^n\ b, b^{2n}) \mid n > 0\}$$

Properties of Visibly Pushdown Transducers

- NFT $<$ VPT $<$ PT
- dVPTs $<$ (functional) VPT
- closed under composition if the output is well-nested
- closed under VPA-lookahead
- functionality is decidable in PTime
- k-valuedness is decidable
- equivalence of functional VPTs is decidable (in PTime of dVPTs)
- decidable typechecking problem (if the output is well-nested)

Properties of Visibly Pushdown Transducers

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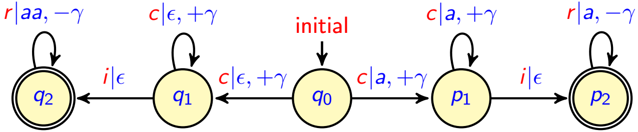
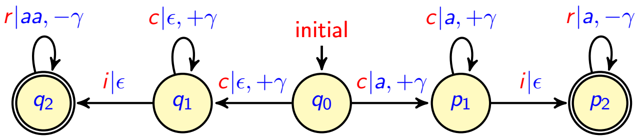
- equivalence of functional VPTs is decidable (in PTime of dVPTs)
- decidable typechecking problem (if the output is well-nested)
- **Open Problems**: equivalence of k-valued VPTs, determinizability
- more details in F. Servais's Phd thesis

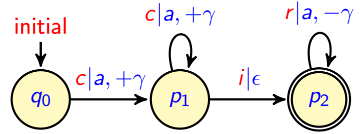
Why is determinizability more difficult?



Why is determinizability more difficult?



It is determinizable by:



but lag increase arbitrarily in (p_1, q_1).

Streamability Problem [F, Gauwin, Reynier, Servais, 11]

Streaming evaluation: avoid the storage of the whole input

Fix a functional (non-deterministic) VPT T.
How much memory is needed to compute T(u) from an input stream u?

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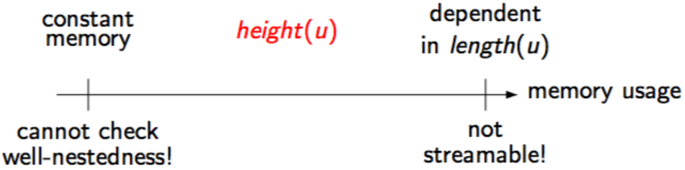

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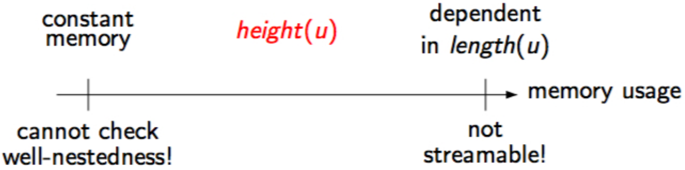

Streamability Problem

Given a VPT T, decide if T defines a transformation that can be evaluated with memory $O(f(height(u))$?

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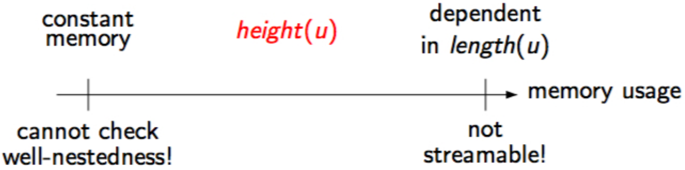

Streamability Problem

Given a VPT T, decide if T defines a transformation that can be evaluated with memory $O(f(height(u)))$?

Decidable in NP for VPTs

Determinizability is too strong

Obs: Deterministic VPTs are always streamable (no output lag)

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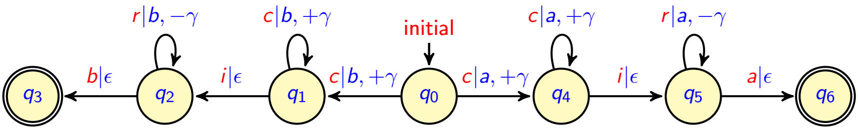
Obs: Deterministic VPTs are always streamable (no output lag)

However: determinizable VPTs $<$ streamable VPTs:

$$R(T) : c^n \ i \ r^n \ \alpha \ \mapsto \alpha^{2n} \quad n > 0$$

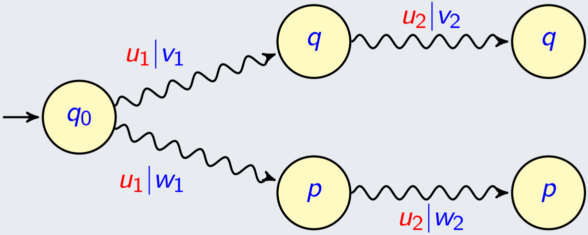


Streamable but not determinizable !

Twinning Property for VPTs

Definition

For all such situations



it is the case that $LAG(v_1, w_1) = LAG(v_1 v_2, w_1 w_2)$.

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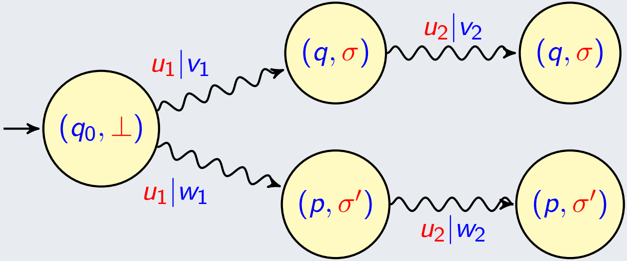

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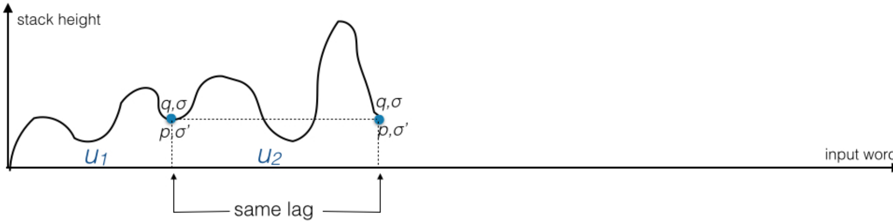

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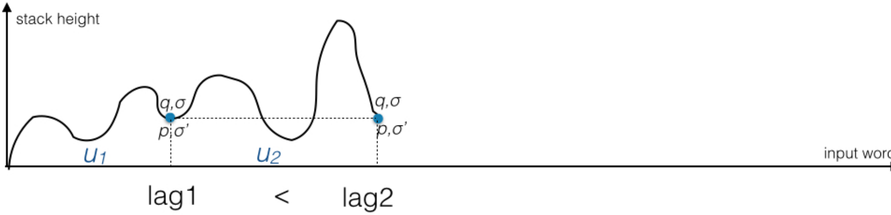

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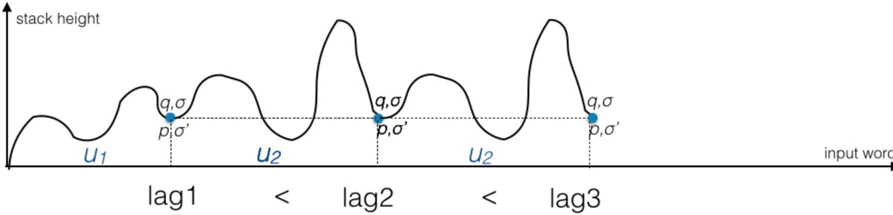

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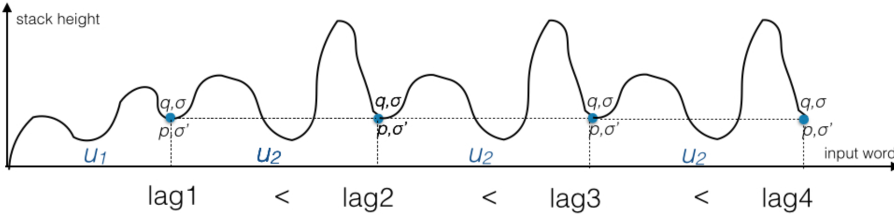

Twinning Property for VPTs

Theorem

Given a functional VPT T, T is streamable iff the twinning property holds.
It can be decided in NPtime.

Twinning Property for VPTs

Theorem

Given a functional VPT T, T is streamable iff the twinning property holds.
It can be decided in NPtime.

- TP is machine-independent: streamable VPTs is class of **transductions**.
- decidability based on reversal-bounded pushdown counter machines
- same result extend to **strongly streamable** (memory depends only on *current height*)

Other tree transducer models

- top-down tree transducers

$$q(f(x_1, \ldots, x_n)) \to C[q_1(x_{i_1}), \ldots, q_p(x_{i_p})]$$

(see TATA[3] book

- macro tree transducers

```
fun q(t1 t2 t3 t4 t)=
if t = a() then
 return F (t1,t2)
else
 if t=g(u,v) then
 return C(q'(t1,t2,u), q''(t3,t4,v))
```

- see Joost Engelfriet and Sebastian Maneth's work

[3] *Tree Automata Techniques and Applications*, tata.gforge.inria.fr

Church Problem

Church Problem (aka Church Synthesis Problem)

Definition (Church 57)

- R a relation, or *requirements*, from a domain D to a domain D'
- synthesize a program P such for all $X \in D$, $(X, P(X)) \in R$.

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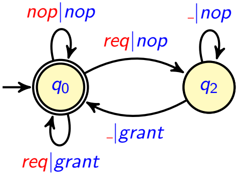
Reactive System Synthesis

Let Σ_{in} and Σ_{out} be to finite alphabets.

- reactive systems continuously react to stimuli produced by some **uncontrollable** environment
- $D = \Sigma_{in}^{\omega}$, $D' = \Sigma_{out}^{\omega}$
- R is a synchronous relation given by a (non-deterministic) symbol-to-symbol Büchi transducer
- P is a Mealy machine (deterministic symbol-to-symbol transducer)

Reactive System Synthesis: Example

- $\Sigma_{in} = \{req, nop\}$
- $\Sigma_{out} = \{grant, nop\}$.
- Requirement R: if there is a request, it must be eventually granted



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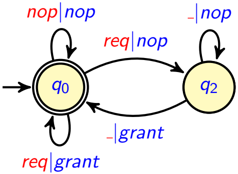
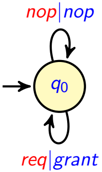
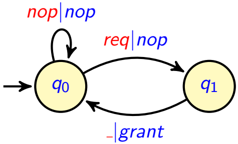


- Possible programs (Mealy machines) that realize R:





Church Game

Definition

- **turn-based** game between two players
- Player *in* chooses input symbols in Σ_{in}
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- they play during an infinite number of rounds.

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Player *out* (Σ_{out}) :

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Player *out* (Σ_{out})	:	o_1	o_2

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Player in (Σ_{in})	:	i_1	i_2	i_3	i_4
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Player in (Σ_{in})	:	i_1	i_2	i_3	i_4	i_5	$\dots$
Player *out* (Σ_{out})	:	o_1	o_2	o_3	o_4	o_5	$\dots$

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Player in (Σ_{in})	:	i_1	i_2	i_3	i_4	i_5	$\dots$
Player *out* (Σ_{out})	:	o_1	o_2	o_3	o_4	o_5	$\dots$

- **Def:** Player *out* wins if $(i_1 i_2 i_3 \dots, o_1 o_2 o_3 \dots) \in R$.

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- Player *in* chooses input symbols in Σ_{in}
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$$\begin{array}{lcllllll} \text{Player in } (\Sigma_{in}) & : & i_1 & i_2 & i_3 & i_4 & i_5 & \dots \\ \text{Player } out \ (\Sigma_{out}) & : & o_1 & o_2 & o_3 & o_4 & o_5 & \dots \end{array}$$

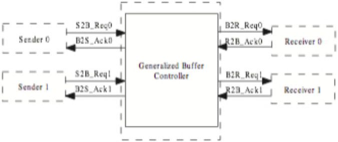
- **Def:** Player *out* wins if $(i_1 i_2 i_3 \dots, o_1 o_2 o_3 \dots) \in R$.
- **Prop:** There exists a program that realizes the requirements R iff Player *out* has a winning strategy.

State of the Art

- reactive system synthesis from ω-regular specifications is decidable (Büchi Landweber 69)
- reactive system synthesis from LTL specifications is 2-ExpTime-c (Pnueli Rosner 89)
- several tools for LTL synthesis:
  - Lily (Jobstmann Bloem 06)
  - Acacia (Filiot Jin Raskin 09)
  - Unbeast (Ehlers 10)
- very active community in game theory for synthesis
  - quantitative games
  - multi-player games
  - stochastic games
  - ...

Acacia Tool

GenBuf spec from IBM




```
################################################
[spec_unit sb_0]
################################################
assume s2b_req0=0;
assume G((s2b_req0=1 * b2s_ack0=0) -> X(s2b_req0=1));
assume G(b2s_ack0=1 -> X(s2b_req0=0));

b2s_ack0=0;
G((s2b_req0=0 * X(s2b_req0=1)) -> X(b2s_ack0=0 * X(F(b2s_ack0=1))));
G((b2s_ack0=0 * X(s2b_req0=0)) -> X(b2s_ack0=0));
G(b2s_ack0=0 + b2s_ack1=0);

################################################
[spec_unit sb_1]
################################################
assume s2b_req1=0;
assume G((s2b_req1=1 * b2s_ack1=0) -> X(s2b_req1=1));
assume G(b2s_ack1=1 -> X(s2b_req1=0));

b2s_ack1=0;
G((s2b_req1=0 * X(s2b_req1=1)) -> X(b2s_ack1=0 * X(F(b2s_ack1=1))));
G((b2s_ack1=0 * X(s2b_req1=0)) -> X(b2s_ack1=0));
G(b2s_ack0=0 + b2s_ack1=0);

################################################
[spec_unit br_0]
################################################
assume r2b_ack0=0;
assume G(b2r_req0=0 -> X(r2b_ack0=0));
assume G(b2r_req0=1 -> X(F(r2b_ack0=1)));

b2r_req0=0;
G(r2b_ack0=1 -> X(b2r_req0=0));
G((b2r_req0=1 * r2b_ack0=0) -> X(b2r_req0=1));
G((b2r_req0=1 * X(b2r_req0=0)) -> X(b2r_req0=0 U (b2r_req0=0 * b2r_req1=
G((b2r_req0=0) + (b2r_req1=0) );
G((s2b_req0=1 + s2b_req1=1) -> X(F(b2r_req0=1 + b2r_req1=1)));

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assume r2b_ack1=0;
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assume G(b2r_req1=1 -> X(F(r2b_ack1=1)));
```

Scalable example

From 1-page long to 4-page long specifications

http://lit2.ulb.ac.be/acaciaplus

How is it related to transducer theory?

- reactive systems are streaming machines
- from a relation R, extract a function f such that:
  1. $dom(R) \subseteq dom(f)$
  2. for all $u \in dom(R)$, $f(u) \in R(u)$.
  3. f is a deterministic symbol-to-symbol transducer
- this problem is known as the uniformization problem in transducer theory
- equivalently, is there a bounded memory (symbol-to-symbol) function f such that $f \subseteq R$ and $dom(R) \subseteq dom(f)$?

Conclusion

Contributions

- finite transducers have good closure and algorithmic properties
- nicely extend to visibly pushdown transducers
- streamability problem $\equiv$ synthesis problem

Open Problems and Future Work

Open problems

- equivalence of k-valued VPTs
- determinizability of VPTs
- extension of streaming results to more expressive transducers, e.g. macro tree transducers
- shift from reactive systems to list processing program synthesis

Publications

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