When is Metric Temporal Logic Expressively Complete?

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Reasoning about time

Timed systems are everywhere:

- Hardware circuits
- Communication protocols
- Cell phones
- Plant controllers
- Aircraft navigation systems
- ...

Want to specify:

"*If I press the brake pedal then the pads will be applied*."

Expressiveness *vs* **Computability**

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LTL has emerged as the definitive temporal logic.

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- As expressive as first order logic [Kamp 68]

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"*If I press the brake pedal then*
the pads will be applied between 0.5ms and 1ms."

Metric Temporal Logic (MTL)

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[Koymans; de Roever; Pnueli $\sim$1990] is LTL with timing constraints added to the temporal modalities

Problem:
How expressive is MTL?

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Depends on the timing constants used...

- With no constants: MTL$=$FO [Kamp 68]
- With integer constants: MTL$\neq$FO [Hirshfeld and Rabinovich 07]
- With rational constants: MTL$=$FO [H., Ouaknine and Worrell 13]

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Temporal models

- A set **MP** of propositions: P, Q, R, ...
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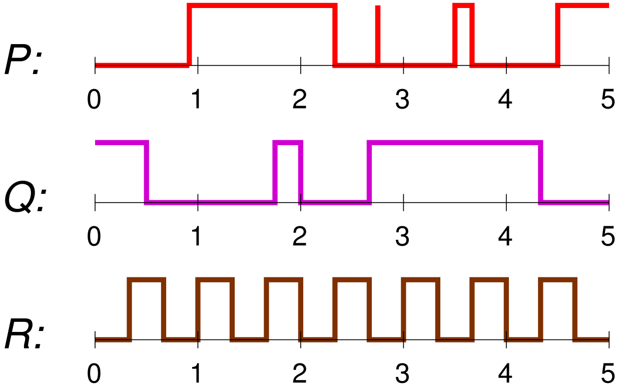
$$f : \mathbb{R} \to 2^{\mathbf{MP}}$$ (flow or signal)



Classic temporal predicate logic

FO($<$): First-order logic with $<$ and monadic predicates for each proposition $P \in \mathbf{MP}$:

$$\varphi ::= x < y \mid P(x) \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \neg\varphi \mid \forall x\, \varphi \mid \exists\, x \varphi$$

For example:

$$\forall x \,.\, \texttt{PEDAL}(x) \rightarrow \exists y \,.\, \big((y > x) \wedge \texttt{BRAKE}(y)\big).$$

Metric Predicate Logic

Given a set $\mathcal{K} \subseteq \mathbb{R}$ of constants we add many unary functions $\{+c : c \in \mathcal{K}\}$ to FO($<$) to model moving c time units into the future.

$$\forall x.\mathtt{PEDAL}(x) \to \exists y.\,(x{+}5 < y < x{+}10) \land \mathtt{BRAKE}(y),$$

a formula of $\mathrm{FO}_{\{5,10\}}$.

Temporal logic: LTL

LTL: Propositional logic with temporal modalities:

$$
\begin{array}{rcll}
\theta & ::= & P \mid \theta_1 \wedge \theta_2 \mid \theta_1 \vee \theta_2 \mid \neg\theta & \\
 & & \mid \mathbf{F}\,\theta & (\theta \text{ occurs in the Future}) \\
 & & \mid \mathbf{G}\,\theta & (\theta \text{ occurs always (Globally)}) \\
 & & \mid \theta_1 \;\mathbf{U}\; \theta_2 & (\theta_1 \text{ holds Until } \theta_2) \\
 & & \mid \mathbf{P}\,\theta & (\theta \text{ occurred in the Past}) \\
 & & \mid \mathbf{H}\,\theta & (\theta \text{ has always occurred (Historically)}) \\
 & & \mid \theta_1 \;\mathbf{S}\; \theta_2 & (\theta_1 \text{ has held Since } \theta_2)
\end{array}
$$

For example,

$$\mathbf{G}\,(\mathtt{PEDAL} \rightarrow \mathbf{F}\,\mathtt{BRAKE})$$

Metric Temporal Logic

$\text{MTL}_{\mathcal{K}}$ = LTL + timing constraints taken from $\mathcal{K}$ on operators:

$$
\begin{array}{rcll}
\theta & ::= & P \mid \theta_1 \wedge \theta_2 \mid \theta_1 \vee \theta_2 \mid \neg\theta & \\
& & \mid \mathbf{F}_I\, \theta & (\theta \text{ occurs in the Future in the interval } I) \\
& & \mid \mathbf{G}_I\, \theta & (\theta \text{ occurs always (Globally) in the interval } I) \\
& & \mid \theta_1\, \mathbf{U}_I\, \theta_2 & (\theta_2 \text{ holds in } I \text{ and Until then } \theta_1 \text{ holds}) \\
& & \mid \mathbf{P}_I\, \theta & \theta \text{ occurred in the Past in the interval } I) \\
& & \mid \mathbf{H}_I\, \theta & (\theta \text{ always occurred in the interval } I) \\
& & \mid \theta_1\, \mathbf{S}_I\, \theta_2 & (\ \theta_2 \text{ held in } I \text{ and } \theta_1 \text{ has held Since})
\end{array}
$$

where I is an interval with end-points in $\mathcal{K}$.

$$\mathbf{G}\,(\texttt{PEDAL} \rightarrow \mathbf{F}_{(5,10)}\,\texttt{BRAKE})$$

Adding time metrics to the models

What sets of constants $\mathcal{K}$?

- Traditional approach: intervals over $\mathbb{Z}$
- Continuous but finitely presentable: intervals over $\mathbb{Q}$
- Intervals over an arbitrary additive subgroup of $\mathbb{R}$...

Adding time metrics to the models

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- Traditional approach: intervals over $\mathbb{Z}$
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Additive subgroup?

- Can easily form integer linear combinations of timing constants.
- Integer linear combinations of $\mathcal{K}$ = Subgroup of $(\mathbb{R}, +)$ generated by $\mathcal{K}$.

Motivation:

- Includes most general case ($\mathcal{K} = \mathbb{R}$)
- Generalizes previous cases ($\mathcal{K} = \mathbb{Z},\ \mathbb{Q},$ or $\{0\}$)
- Can be used to model multiple independent asynchronous timing systems (e.g. $\mathbb{Z}[\sqrt{2}]$)

Main result

Theorem

$MTL_{\mathcal{K}}$ = $FO_{\mathcal{K}}$ if and only if $\mathcal{K}$ is dense.

Proof: “Only if”

Lemma

If $\mathcal{K}$ is a non-dense additive subgroup of $\mathbb{R}$ then $\mathcal{K} = \epsilon\mathbb{Z}$ for some $\epsilon \in \mathbb{R}$.

Proof: “If”

1. Use “metric separation” to reduce to bounded formulas.
2. Use a normal form for $\text{FO}_{\mathcal{K}}$ formulas to remove $+c$ functions.
3. Use denseness of $\mathcal{K}$ to express LTL statements restricted to an interval.

Separation

Expressive completeness of LTL can be proven by separating formulas into *past*, *present*, and *future*.

Separation does not hold in the quantitative setting.

For example,

$$\mathbf{G}(\texttt{BRAKE} \rightarrow \mathbf{P}_{(5,10)}\texttt{PEDAL})$$

General quantitative separation

Given a constant $c > 0$, a metric temporal formula is:

- pure c-distant past if it is invariant on flows that agree on $(-\infty, -c)$
- pure c-distant future if it is invariant on flows that agree on (c, ∞)
- bounded if there is an N such that it is invariant on all flows that agree on $(-N, N)$

A temporal logic with constants from $\mathcal{K}$ is generally metrically separable if every formula is equivalent, for some $c \in \mathcal{K}_{>0}$, to a boolean combination of pure c-distant past, pure c-distant future and bounded formulas.

Lemma

$MTL_{\mathcal{K}}$ is generally metrically separable for non-trivial $\mathcal{K}$.

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A temporal logic with constants from $\mathcal{K}$ is generally metrically separable if every formula is equivalent, for some $c \in \mathcal{K}_{>0}$, to a boolean combination of pure c-distant past, pure c-distant future and bounded formulas.

Corollary

$MTL_{\mathcal{K}}$ = $FO_{\mathcal{K}}$ iff $MTL_{\mathcal{K}}$ can express all bounded $FO_{\mathcal{K}}$ formulas.

Removing the unary functions

First remove unary functions from monadic predicates by introducing new predicates: e.g. $P(x+5) = P_5(x)$

$$
\begin{aligned}
\varphi(x) &= \exists y \in (x, x+1)\, \exists z \in (y, y+\sqrt{2}) \ldots \\
&= \exists y \in (x, x+1)\, \big(\exists z \in (y, x+1) \ldots \\
&\qquad \vee \exists z \in (x+1, y+\sqrt{2}) \ldots \big) \\
&= \exists y \in (x, x+1)\, \big(\exists z \in (y, x+1) \ldots \\
&\qquad \vee \exists z' \in (x+1-\sqrt{2}, y) \ldots \big) \\
\varphi(x_0, x_1, x_2) &= \exists y \in (x_1, x_2)\, \big(\exists z \in (y, x_2) \ldots \vee \exists z' \in (x_0, y) \ldots \big)
\end{aligned}
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Corollary

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Removing the unary functions

Move the remaining unary functions to the free variable

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Replace the "milestones" ($\{x+1-\sqrt{2}, x, x+1\}$) with new variables to obtain a FO($<$) formula.

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$MTL_{\mathcal{K}} = FO_{\mathcal{K}}$ iff $MTL_{\mathcal{K}}$ can express all bounded $FO_{\{0\}}$ formulas.

Removing the unary functions

Use a model-theoretic argument to break this into formulas on the intervals $\{x_0\}, (x_0, x_1), \{x_1\}, \ldots$

$$\begin{aligned}
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$MTL_{\mathcal{K}}$ = $FO_{\mathcal{K}}$ iff $MTL_{\mathcal{K}}$ can express all bounded $FO_{\{0\}}$ formulas.

Failure of Kamp's theorem

$\mathrm{MTL}_{\mathbb{Z}}$ is unable to express:

"*P occurs twice in the next time interval.*"

In $\mathrm{FO}_{\mathbb{Z}}$:

$$\varphi(z) = \exists x.\exists y.(z < x < z +) \wedge (z < y < z +) \wedge \mathrm{P}(x) \wedge \mathrm{P}(y).$$

In $\mathrm{MTL}_{\mathbb{Z}}$:

$$(\mathbf{F}_{(0,1)}\mathrm{P} \wedge \mathbf{F}_{(1,2)}\mathrm{P}) \quad \vee$$
$$\mathbf{F}_{=2}\Big(\mathbf{P}_{(0,1)}\big(\mathrm{P} \wedge \mathbf{P}_{(0,1)}\mathrm{P}\big)\Big)$$

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Failure of Kamp's theorem

$\mathsf{MTL}_{\mathbb{Z}}$ is able to express:

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In $\mathsf{FO}_{\mathbb{Z}}$:

$$\varphi(z) = \exists x. \exists y. (z < x < z + 2) \wedge (z < y < z + 2) \wedge \mathtt{P}(x) \wedge \mathtt{P}(y).$$

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$$\begin{aligned} \varphi \quad = \quad & \mathbf{F}_{(0,1)}\big(\mathtt{P} \wedge \mathbf{F}_{(0,1)}\mathtt{P}\big) \quad \vee \\ & \big(\mathbf{F}_{(0,1)}\mathtt{P} \wedge \mathbf{F}_{(1,2)}\mathtt{P}\big) \quad \vee \\ & \mathbf{F}_{=2}\Big(\mathbf{P}_{(0,1)}\big(\mathtt{P} \wedge \mathbf{P}_{(0,1)}\mathtt{P}\big)\Big) \end{aligned}$$

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Adding granularity

$$\begin{aligned}\varphi \quad = \quad & \mathbf{F}_{(0,1)} \ \big(\mathtt{P} \land \mathbf{F}_{(0,1)} \ \mathtt{P}\big) \quad \lor \\ & (\mathbf{F}_{(0,1)} \ \mathtt{P} \land \mathbf{F}_{(1,2)} \ \mathtt{P}) \quad \lor \\ & \mathbf{F}_{=2}\Big(\mathbf{P}_{(0,1)} \ \big(\mathtt{P} \land \mathbf{P}_{(0,1)} \ \mathtt{P}\big)\Big)\end{aligned}$$

Corollary

"P occurs twice in the next time interval" is expressible in $MTL_{\mathbb{Q}}$.

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$$\begin{aligned}\varphi \quad = \quad & \mathbf{F}_{(0,\frac{1}{2})}\ \left(\mathtt{P} \wedge \mathbf{F}_{(0,\frac{1}{2})}\ \mathtt{P}\right) \quad \vee \\ & (\mathbf{F}_{(0,\frac{1}{2})}\ \mathtt{P} \wedge \mathbf{F}_{(\frac{1}{2},1)}\ \mathtt{P}) \quad \vee \\ & \mathbf{F}_{=1}\left(\mathbf{P}_{(0,\frac{1}{2})}\ \left(\mathtt{P} \wedge \mathbf{P}_{(0,\frac{1}{2})}\ \mathtt{P}\right)\right)\end{aligned}$$

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"P occurs twice in the next time interval" is expressible in MTL$_{\mathbb{Q}}$.

Counting is all you need...

Theorem

$MTL_{\mathbb{Z}}$ with counting modalities has the same expressive power as $FO_{\mathbb{Z}}$.

Corollary

$MTL_{\mathcal{K}}$ can express any bounded LTL formula if $\mathcal{K}$ is dense and non-trivial

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A true extension of Kamp's theorem

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