The Expressiveness of Metric Temporal Logic

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LICS, 27 June 2013

Motivation

Timed systems are everywhere.

Want to specify:

"*If the brake pads were applied then the pedal was pushed*."

$\mathbf{G}\,(\mathrm{BRAKE} \rightarrow \mathbf{P}\,\mathrm{PEDAL})$

Expressiveness *vs* Computability

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Expressiveness *vs* Computability

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Quantitative logics can be more practical.

Want to specify:

"*If the brake pads were applied then the pedal was pushed between 0.5ms and 1ms ago*."

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Predicate Logics → **Expressiveness** ***vs*** **Computability** ← **Temporal Logics**

Motivation

In the qualititative setting, LTL is all you need:

Theorem (Kamp 1968)

LTL is as expressive as FO.

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Is there an analogue of Kamp's Theorem in the quantitative setting?

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Metric Temporal Logic

Bounded formulas

Metric separation

Corollaries, Conclusions and Continuations

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Temporal models

- A set **MP** of propositions: $P, Q, R, \ldots$
- Continuous time model: $\mathbb{R}$

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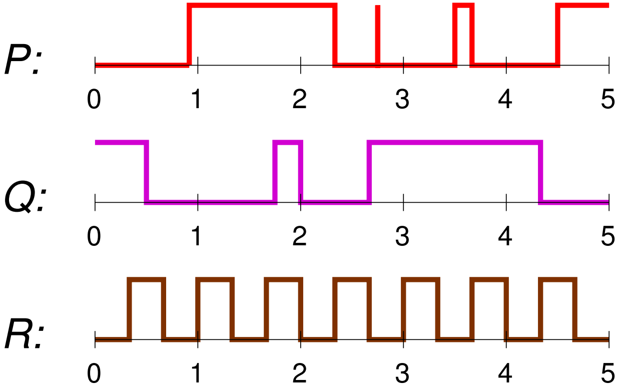

Classic temporal predicate logic

FO(<): First-order logic with $<$ and monadic predicates for each proposition $P \in \mathbf{MP}$:

$$\varphi ::= x < y \mid P(x) \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \neg\varphi \mid \forall x\, \varphi \mid \exists x \varphi$$

For example:

$$\forall x\,.\,\mathsf{BRAKE}(x) \rightarrow \exists y\,.\,\big((y < x) \wedge \mathsf{PEDAL}(y)\big).$$

Temporal logic: LTL

Linear Temporal Logic (LTL): Propositional logic with temporal modalities:

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\begin{aligned}
\theta \quad ::= \quad & P \mid \theta_1 \wedge \theta_2 \mid \theta_1 \vee \theta_2 \mid \neg\theta \\
& \mid \mathbf{F}\,\theta && (\theta \text{ occurs in the Future}) \\
& \mid \mathbf{G}\,\theta && (\theta \text{ occurs always (Globally)}) \\
& \mid \theta_1\,\mathbf{U}\,\theta_2 && (\theta_1 \text{ holds Until } \theta_2) \\
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For example,

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LTL is subsumed by FO($<$), for example

$$P\,\mathbf{U}\,Q \quad \equiv \quad \exists x\,(Q(x) \wedge \forall y\,.\,(y < x) \rightarrow P(y)).$$

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Kamp's theorem

Theorem (Kamp 1968, GPSS 1980)

LTL is as expressive as FO($<$).

Metric Temporal Logic

Metric Temporal Logic (MTL) [Koymans; de Roever; Pnueli ∼1990] is a central quantitative specification formalism for timed systems.

MTL = LTL + timing constraints on operators:

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Formally,

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\theta & ::= & P \mid \theta_1 \wedge \theta_2 \mid \theta_1 \vee \theta_2 \mid \neg\theta & \\
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where I is an interval of $\mathbb{Q}$.

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where I is an interval of $\mathbb{Q}$.

Metric Predicate Logic

We add many unary functions $\{+q : q \in \mathbb{Q}\}$ to FO($<$) to model moving q time units into the future.

$$\forall x.\mathsf{BRAKE}(x) \to \exists y.\,(x - 10 < y < x - 5) \wedge \mathsf{PEDAL}(y),$$

a formula of FO($<$,$+q$).

With integral constraints, FO($<$,$+1$) suffices.

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A non-extension of Kamp's theorem

Theorem (Hirshfeld & Rabinovich 2007)

MTL with integer end-points is strictly less expressive than FO($<$,$+1$) over $\mathbb{R}$, even with infinitely many additional modal operators of bounded quantifier depth.

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Main result

Theorem

MTL has the same expressive power as FO($<$,$+q$).

Metric Temporal Logic

Bounded formulas

Metric separation

Corollaries, Conclusions and Continuations

Bounded formulas

Lemma

MTL can express all bounded FO($<$,$+1$)-formulas.

Bounded formulas

MTL with integer endpoints is unable to express:

"*P occurs twice in the next time interval.*"

In FO($<, +1$):

$$\varphi(z) = \exists x . \exists y . (z < x < y < z + \ ) \wedge \mathrm{P}(x) \wedge \mathrm{P}(y).$$

In MTL:

$$\left(\mathbf{F}_{(0,1)}\mathrm{P} \wedge \mathbf{F}_{(1,2)}\mathrm{P}\right) \quad \vee$$
$$\mathbf{F}_{=2}\left(\mathbf{P}_{(0,1)}\left(\mathrm{P} \wedge \mathbf{P}_{(0,1)}\mathrm{P}\right)\right)$$

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MTL with integer endpoints is able to express:

"*P occurs twice in the next two time intervals.*"

In FO($<$, $+1$):

$$\varphi(z) = \exists x.\exists y.(z < x < y < z + 2) \wedge \mathsf{P}(x) \wedge \mathsf{P}(y).$$

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Adding granularity

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Corollary

"P occurs twice in the next time interval" is expressible in MTL with rational end-points.

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Metric Temporal Logic

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Metric separation

Corollaries, Conclusions and Continuations

Separation of temporal logics

Separability is a key property for showing LTL and FO have the same expressive power.

A temporal logic formula is

- pure past if it is invariant on flows that agree on the past
- pure present if is invariant on flows that agree on the present
- pure future if is invariant on flows that agree on the future

A temporal logic is separable if all its formulas are equivalent to a boolean combination of pure past, present and future formulas.

$$\mathbf{G}(\mathsf{BRAKE} \rightarrow \mathbf{P}\,\mathsf{PEDAL})$$

$$\equiv \mathbf{P}\,\mathsf{PEDAL} \;\vee\; (\neg\mathsf{BRAKE}\;\mathbf{U}\;\mathsf{PEDAL}) \;\vee\; \mathbf{G}(\neg\mathsf{BRAKE})$$

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Separation of temporal logics

Lemma

LTL is separable.

Theorem (Gabbay 1981)

A temporal logic is expressively complete if and only if it is separable.

Corollary (Kamp's theorem)

LTL is expressively complete.

Quantitative separation

Separation does not hold in the quantitative setting.

For example,

$$\mathbf{G}(\mathsf{BRAKE} \rightarrow \mathbf{P}_{(5,10)}\mathsf{PEDAL})$$

Quantitative separation

A metric temporal formula is:

- pure distant past if it is invariant on flows that agree on $(-\infty, -1)$
- pure distant future if it is invariant on flows that agree on $(1, \infty)$
- bounded if there is an N such that it is invariant on all flows that agree on $(-N, N)$

A temporal logic is metrically separable if every formula is equivalent to a boolean combination of pure distant past, pure distant future and bounded formulas.

Lemma

MTL is metrically separable.

$$\mathbf{G}(\mathrm{BRAKE} \rightarrow \mathbf{P}_{(5,10)}\mathrm{PEDAL}) = \mathbf{G}_{(0,11]}(\mathrm{BRAKE} \rightarrow \mathbf{P}_{(5,10)}\mathrm{PEDAL}) \wedge \mathbf{G}_{(11,\infty)}(\mathrm{BRAKE} \rightarrow \mathbf{P}_{(5,10)}\mathrm{PEDAL})$$

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An extension of Gabbay's theorem

Corollary

MTL can express FO($<$,$+1$) if and only if it can express all bounded formulas.

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Conclusions and further work

- First generalization of Kamp's theorem to the quantitative setting.
- Generalization of Gabbay's theorem to the quantitative setting.

Recent results (CSL 2013):

- Other sets of constants (e.g. $\mathbb{R}$, $\mathbb{Z}[\sqrt{2}]$).
- Expressive completeness for MTL with counting.

Still to do:

- Cost of expressibility.
- Better generalization of Gabbay's Theorem.
- Extension to more expressive metric temporal logics.

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